Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

A Novel Finite Time Sliding Mode Control for Robotic Manipulators ⋆

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Abstract: A novel robust control law with finite time convergence for rigid robotic manipulators is proposed in this paper. The whole control process is divided into two phases, i.e., the error correction phase and the steady tracking phase. Firstly, a novel time-varying sliding mode control (TVSMC) method is developed in the first phase to ensure the tracking error converge to zero at the desired time. Subsequently, the nonsingular terminal sliding mode control (NTSMC) technique is employed to keep the tracking error maintaining zero during the second phase. The associated control commands are free from singularity and convenient for practical implementation. Besides, the convergence rate can be tuned via appropriate parameter adjustment. By the virtue of global sliding mode, the system is global robust against the external disturbances and parametric uncertainties. Numerical simulations are presented to validate the effectiveness of the proposed control law.

Keywords: robotic manipulators, finite time, time-varying sliding mode control (TVSMC), nonsingular terminal sliding mode control (NTSMC), global robust.

1. INTRODUCTION

Robotic manipulators have been widely applied in modern industry owing to their advantages such as higher load-to-weight ratio, less energy consumption and higher speed (Wang et al. (2011)). In practical application, however, a rigid robotic manipulator is a kind of uncertain nonlinear multi-input multi-output (MIMO) system which suffers from unmodeled dynamics, unstructured uncertainties and external disturbances, and thus it is not appropriate to derive the control strategy completely on the basis of the plant model. As a consequence, the controller design for a robotic manipulator is really a challenging issue.

There are basically two philosophies for controlling a robotic manipulator with nonlinear and uncertain behaviors (Khan et al. (2012)), i.e., adaptive control(Berghuis et al. (1993), Ortega and Spong (1989), Slotine and Li (1989)) and robust control. To achieve the best performance, the former tries to identify the uncertain parameters of the system while the latter possesses fixed structure which is insensitive to the parameter variation. The robust control technique has been widely studied in the past decades because of the ease of design due to less information demand. As an elementary approach to robust control, the sliding mode control (SMC) methodology can offer many good properties, such as insensitivity to parameter variation, external disturbance rejection, and fast dynamic response (Slotine and Li (1991)). Traditional SMC can only guarantee asymptotic stability which implies that the tracking error converge to zero as time goes to infinity. In real time implementation, however, infinite time convergence may not be enough.

Finite time control always provides superior properties, such as faster convergence rate, higher accuracy, better robustness against uncertainties and disturbance rejection property (Wang et al. (2012), Du et al. (2011)). To achieve finite time convergence of uncertain dynamic systems, the terminal sliding mode control (TSMC) is developed in Wu and Yu (1998). This method is further extended in Yu and Man (2002) to achieve a faster convergence rate. However, singularity may be experienced during the implementation of TSMC. To overcome this drawback, a nonsingular terminal sliding mode control (NTSMC) technique is presented in Feng et al. (2002). This strategy is able to get rid of the singularity without adding any extra procedures. Afterwards, this method is further investigated in Yu et al. (2005) to derive a new class of continuous TSM controllers. In recent years, artificial intelligence theory has been integrated into the TSMC design. The fuzzy logic controller is incorporated into TSMC in Li and Huang (2010) to retain the advantages of TSMC while alleviating the chattering phenomenon. In addition, a new adaptive TSMC is presented in Neila and Tarak (2011) to estimate the bounds of uncertainties, and meanwhile, eliminating the chattering effect.

The finite time control problem has seen a large amount of attention, in contrast there has been a negligible amount of work performed towards solving the fixed time convergence control problem. As far as the authors know, the only published work coping with this issue can be found in Laghrouche et al. (2007). The associated controller consist of two parts, the first part is the integral sliding mode (IS-M) controller to counteract the lumped uncertainty while

⋆ This study was co-supported by National Science Foundation of China (No. 11372034).

the other part is a kind of optimal feedback control law to guarantee a desired finite time convergence. However, this control law is just designed for single-input single-output (SISO) systems.

This paper investigates the finite time control problem for the rigid robotic manipulators which belongs to the category of MIMO systems. A novel robust control strategy which is synthesized with time-varying sliding mode control (TVSMC) theory and the NTSMC technique is introduced. This strategy features in three characteristics: First, the time convergence can be chosen in advance which implies the tracking error would converge to zero at a desired finite time. Second, the convergence rate can be tuned by parameter adjusting. Third, by the virtue of global sliding mode, the controlled system is global robust against parametric uncertainty and external disturbance. In addition, it should be noted that the zero velocity error assumption at the initial time is not required, which is a necessity in some previous work such as Bartoszewicz (1996) and Cong et al. (2012).

2. MATHEMATICAL MODEL AND PROBLEM STATEMENT

2.1 Mathematical model of rigid robotic manipulators

Consider a rigid r-link robotic manipulator described by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u + u_d \tag{1}$$

where $q \in \mathbb{R}^r$ is the vector of joint angular position, $M(q) \in \mathbb{R}^{r\times r}$ is the symmetric positive definite inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{r\times r}$ is the Coriolis and centrifugal forces matrix, $G(q) \in \mathbb{R}^r$ is the gravitational torque, $u \in \mathbb{R}^r$ is the vector of input torque, and $u_d \in \mathbb{R}^r$ is the vector of bounded external disturbance.

The uncertainties of the rigid robotic manipulator is assumed, i.e.:

$$M(q) = M_0(q) + \Delta M(q)$$

$$C(q,\dot{q}) = C_0(q,\dot{q}) + \Delta C(q,\dot{q})$$

$$G(q) = G_0(q) + \Delta G(q)$$

where $M_0(q)$, $C_0(q,\dot{q})$, $G_0(q)$ represent the nominal terms and $\Delta M(q)$, $\Delta C(q,\dot{q})$, $\Delta G(q)$ stand for the uncertain terms. Then, the system dynamics in (1) can be rewritten in the following form

$$M_0(q)\ddot{q} + C_0(q,\dot{q})\dot{q} + G_0(q) = u + d \tag{2}$$

where d is regarded as the lumped uncertainty which can be expressed by

$$d = u_d - \Delta M(q)\ddot{q} - \Delta C(q,\dot{q})\dot{q} - \Delta G(q) \tag{3}$$

It can be observed from (2) that the lumped uncertainty is matched to the controlled system. For further analysis, the following assumptions are made:

$$||M_0^{-1}(q)|| \leq \rho_1, \quad ||d|| \leq \rho_2 + \rho_3||q|| + \rho_4||\dot{q}||$$

where ρ_1, ρ_2, ρ_3, ρ_4 are positive numbers. In the later discussions, for the sake of notational clarity, $M(q)$, $C(q,\dot{q})$, $G(q)$, $M_0(q)$, $C_0(q,\dot{q})$ and $G_0(q)$ are replaced by M, C, G, M_0, C_0 and G_0, respectively.

2.2 Problem statement

The finite time tracking control problem of the system in (2) is addressed in this paper. Assume that $q_d \in \mathbb{R}^r$ is the desired angular trajectory and $\dot{q}_d$ is the derivative of q_d. The control objective is to design a controller u for the robotic manipulator, such that the tracking errors of the angular position q and its derivation $\dot{q}$ can converge to zero at a predetermined time t_f in the presence of external disturbance as well as parametric uncertainty. To sum up, a controller needs to be derived such that

$$q - q_d = \dot{q} - \dot{q}_d = 0, \quad t \geq t_f \tag{4}$$

3. MAIN RESULTS

In this section, a novel finite time control strategy on the basis of SMC is proposed. The whole control action is divided into two phases, i.e., the error correction phase and the steady tracking phase. A novel TVSMC approach is first designed for the first phase to drive the tracking errors to zero at a desired finite time. Then, the NTSMC methodology is employed in the second phase to keep the tracking errors at zero for the rest of the time.

3.1 A novel TVSMC algorithm

Firstly, the tracking angular error and tracking velocity error are defined as follows:

$$\tilde{q} = q - q_d, \quad \dot{\tilde{q}} = \dot{q} - \dot{q}_d \tag{5}$$

Then, a new variable is designed as

$$\sigma = (t_f - t)\dot{\tilde{q}} + n(1 - \frac{1}{e^{\tilde{q}}}) \tag{6}$$

where n is a positive constant and t_f is the desired time at which the tracking errors converge to zero.

Theorem 1. Suppose that $0 \leq t_1 < t_f$ and $n > 1$. If σ remains zero from t_1 onwards, $\tilde{q}$ and $\dot{\tilde{q}}$ will both go to zero exactly at the time $t = t_f$.

Proof. From $\sigma = \mathbf{0}$, one gets

$$\dot{\tilde{q}} = \frac{d\tilde{q}}{dt} = -\frac{e^{\tilde{q}} - 1}{e^{\tilde{q}}}\frac{n}{t_f - t} \tag{7}$$

Rearranging variables, (7) can be rewritten as

$$\frac{e^{\tilde{q}}}{e^{\tilde{q}} - 1}d\tilde{q} = -\frac{n}{t_f - t}dt \tag{8}$$

Assume that the initial state of (8) is $(t_1, \tilde{q}_1)$. Then, integrating (8) from $(t_1, \tilde{q}_1)$ to $(t, \tilde{q})$ yields

$$\int_{\tilde{q}_1}^{\tilde{q}} \frac{e^{\tilde{q}}}{e^{\tilde{q}} - 1}d\tilde{q} = -\int_{t_1}^{t} \frac{n}{t_f - t}dt$$

After some algebra, the following result can be derived

$$e^{\tilde{q}} = 1 + \frac{e^{\tilde{q}_1} - 1}{(t_f - t_1)^n}(t_f - t)^n$$

As a consequence, the analytical expression of $\tilde{q}$ can be obtained

$$\tilde{q} = ln(1 + b(t_f - t)^n) \tag{9}$$

where $b = \frac{e^{\tilde{q}_1} - 1}{(t_f - t_1)^n} \in \mathbb{R}^r$ is a constant vector. Then, differentiating $\tilde{q}$ with respect to t, yielding

$$\dot{\tilde{q}} = \frac{-bn(t_f - t)^{n-1}}{1 + b(t_f - t)^n} \tag{10}$$

From (9) and (10), it can be easily concluded that both $\tilde{q}$ and $\dot{\tilde{q}}$ will converge to zero at $t = t_f$ as long as the value of n is larger than 1.
The proof is completed.

The above theorem reveals that the fixed time convergence can be guaranteed as long as $\sigma = \mathbf{0}$ is reached at some finite amount of time before t_f. For $t > t_f$, however, the steady tracking can not be realized by constraining σ to zero. Such conclusion can be drawn from (9) and (10) which imply the tracking errors would keep increasing from t_f onwards.

Note that at the time when the control process is initiated, the tracking errors, i.e., $\tilde{q}(0)$ and $\dot{\tilde{q}}(0)$, are generally such that $\sigma = \mathbf{0}$ is not satisfied. As a consequence, a reasonable controller u requires to be designed such that σ can be directed toward and constrained at zero.

Such design objective can be achieved by integrating a time-varying term into (6) and the associated time-varying sliding mode function is developed as

$$S_1 = \sigma + \alpha(t) \tag{11}$$

where $\alpha(t)$ is the time-varying function with the initial value α_0 and final value α_f satisfying the following relations:

$$\alpha_0 = -\sigma(0), \quad \alpha_f = \mathbf{0}, \quad \dot{\alpha}_f = \mathbf{0} \tag{12}$$

As to the selection of $\alpha(t)$, different expressions exist in the published literature. Here, the following truncated function is employed (Bartoszewicz (1995, 1996)).

$$\alpha(t) = \begin{cases} At^2 + Bt + D & t \le T \\ \mathbf{0} & t > T \end{cases} \tag{13}$$

where T is the switching time. In accordance with (12), the parameter vectors A, B, $D \in \mathbb{R}^r$ can be determined as follows.

$$D = -\sigma(0), \quad A = \frac{D}{T^2}, \quad B = -2AT \tag{14}$$

Theorem 2. For the system characterized by (2), by choosing the TVSMC law with the form of (15):

$$\begin{aligned} u = \frac{M_0}{t_f - t}\Bigg[&\dot{\tilde{q}} + (t_f - t)\ddot{q}_d - \frac{n\tilde{q}}{e^{\tilde{q}}} \\ &- \begin{cases} 2At + B & t \le T \\ 0 & t > T \end{cases}\Bigg] + C_0\dot{q} + G_0 - \eta M_0 sgn(S_1), \\ &\eta > ||{M_0}^{-1}d||_{max}, \quad 0 < T < t_f, \quad n > 1 \end{aligned} \tag{15}$$

the tracking errors $\tilde{q}$ and $\dot{\tilde{q}}$ will both converge to zero as the time goes to t_f.

Proof. Defining the following positive definite Lyapunov function:

$$V_1 = \frac{1}{2}S_1^T S_1 \tag{16}$$

The time derivative of V_1 can be found as

$$\begin{aligned} \dot{V}_1 &= S_1^T \dot{S}_1 \\ &= S_1^T\Bigg[- \dot{\tilde{q}} + (t_f - t)M_0^{-1}(u + d - C_0\dot{q} - G_0) \\ &\quad - (t_f - t)\ddot{q}_d + \frac{n\dot{\tilde{q}}}{e^{\tilde{q}}} + \begin{cases} 2At + B & t \le T \\ 0 & t > T \end{cases}\Bigg] \end{aligned} \tag{17}$$

Then, substituting the control law (15) into (17), one gets

$$\begin{aligned} \dot{V}_1 &= S_1^T(t_f - t)M_0^{-1}(-\eta M_0 sgn(S_1) + d) \\ &\le -(t_f - t)(\eta - ||{M_0}^{-1}d||_{max})|S_1| \le \mathbf{0} \end{aligned} \tag{18}$$

Apparently, for any $S_1(t) \in \mathbb{R}^r$, V_1 is nonincreasing. As a consequence, one gets $V_1(t) \le V_1(0)$. Since $S_1(0) = \mathbf{0}$, it can be easily drawn that $V_1(t) \le \mathbf{0}$. On the other hand, according to (16), we have $V_1(t) \ge \mathbf{0}$. From the above analysis, we can conclude that $V_1 \equiv \mathbf{0}$ for $t \in [0, t_f]$.

Consider the time-varying term $\alpha(t)$ reaches zero at $t = T$, the value of σ will maintain zero from the time T onwards. According to *Theorem 1*, the tracking errors $\tilde{q}$ and $\dot{\tilde{q}}$ will both go to zero as the time goes to t_f.
The proof is completed.

Remark 3. The choices of T and t_f, from a theoretical point of view, are not limited provided that $0 < T < t_f < \infty$. It should be noted, however, high control amplitude may be encountered if the values of T and t_f are set too small. As a result, the physical limits should be taken into consideration in practical implementation.

Remark 4. It can be easily observed that the term $t_f - t$ appears in the denominator of the control law (15), which would give rise to a singularity at the time $t = t_f$. Since the value of $\alpha(t)$ will turn to zero after T, the associated control effort will changed into the following form:

$$u = M_0\Big(\ddot{q}_d + (1 - \frac{n}{e^{\tilde{q}}})\frac{\dot{\tilde{q}}}{t_f - t}\Big) + C_0\dot{q} + G_0 - \eta M_0 sgn(S_1) \tag{19}$$

Note that σ maintains zero from T to t_f, according to *Theorem 1*, the system state $\dot{\tilde{q}}$ possesses the analytic solution as expressed in (10), which, on substitution in (19), one gets

$$\begin{aligned} u = &M_0\Big(\ddot{q}_d - (1 - \frac{n}{e^{\tilde{q}}})\frac{bn(t_f - t)^{n-2}}{1 + b(t_f - t)^n}\Big) \\ &+ C_0\dot{q} + G_0 - \eta M_0 sgn(S_1) \end{aligned} \tag{20}$$

Consequently, the singular problem can be avoided by setting $n > 2$.

Remark 5. Since the trajectories of the tracking errors, i.e., $\tilde{q}$ and $\dot{\tilde{q}}$, are expressed in (9) and (10) as the time goes close to t_f. The following results can be easily achieved:

$$\begin{aligned} &\lim_{t \to t_f} \tilde{q} = \lim_{t \to t_f} ln(1 + b(t_f - t)^n) = b(t_f - t)^n \\ &\lim_{t \to t_f} \dot{\tilde{q}} = \lim_{t \to t_f} \frac{-bn(t_f - t)^{n-1}}{1 + b(t_f - t)^n} = -bn(t_f - t)^{n-1} \end{aligned} \tag{21}$$

Consequently, $\tilde{q}$ and $\dot{\tilde{q}}$ perform certain behaviors as $t \to t_f$. Besides, it can be further concluded that the convergence rate can be tuned by adjusting the value of n. The larger n is, the faster convergence rate will be.

3.2 NTSMC strategy

By adopting the TVSMC algorithm presented in section 3.1, the tracking errors are able to converge to zero at the desired time t_f. However, they can not be kept on zero for $t > t_f$ due to the inherent properties of this method. Therefore, the NTSMC strategy is proposed in this section to fulfill this requirement.

The non-singular terminal sliding function is described as follows (Feng et al. (2002)):

$$S_2 = \tilde{q} + K|\dot{\tilde{q}}|^{p_1/p_2} sgn(\dot{\tilde{q}}) \quad (22)$$

where $K > 0$ and p_1 and p_2 are positive odd integers, which satisfy $p_2 < p_1 < 2p_2$. For the sake of notational clarity, we will denote p_1/p_2 by β. Then, the time derivative of S_2 is given by:

$$\begin{aligned} \dot{S}_2 &= \dot{\tilde{q}} + K\beta|\dot{\tilde{q}}|^{\beta-1}\ddot{\tilde{q}} \\ &= \dot{\tilde{q}} + K\beta|\dot{\tilde{q}}|^{\beta-1}\big(M_0^{-1}(u + d - C_0\dot{q} - G_0) - \ddot{q}_d\big) \end{aligned} \quad (23)$$

Since $\tilde{q}(t_f) = \dot{\tilde{q}}(t_f) = \mathbf{0}$, it can be obtained from (22) and (23) that both S_2 and $\dot{S}_2$ would be zero at that instant. Besides, consider (11), it is obvious that $S_1(t_f) = \mathbf{0}$. The derivative of S_1 can be obtained as

$$\dot{S}_1 = -\dot{\tilde{q}} + (t_f - t)\ddot{\tilde{q}} + \frac{n\dot{\tilde{q}}}{e^{\tilde{q}}} + \begin{cases} 2At + B & t \le T \\ 0 & t > T \end{cases}$$

From the above equation, we have $\dot{S}_1(t_f) = \mathbf{0}$. Consequently, it can be concluded that the transition between the two sliding surfaces (i.e. S_1 and S_2) are smooth.

Theorem 6. For the system in (2), by adopting the TVSMC algorithm in (15) for $t \in [0, t_f]$ and the NTSMC controller presented in (24) for $t > t_f$,

$$\begin{aligned} u = {} & M_0\Big(\ddot{q}_d - \frac{\dot{\tilde{q}}}{K\beta|\dot{\tilde{q}}|^{\beta-1}}\Big) + C_0\dot{q} + G_0 \\ & - kM_0 sgn(S_2), \quad k > ||{M_0}^{-1}d||_{max} \end{aligned} \quad (24)$$

the following conclusions can be drawn:
(1) The closed loop system is global robust against the lump uncertainty;
(2) The values of $\tilde{q}$ and $\dot{\tilde{q}}$ will converge to zero at t_f and maintain thereafter.

Proof. Let us consider the following positive defined Lyapunov function:

$$V_2 = \frac{1}{2}S_2^T S_2 \quad (25)$$

Differentiate V_2 with respect to t and substitute (24) into the result, yielding:

$$\begin{aligned} \dot{V}_2 &= S_2^T \dot{S}_2 \\ &= S_2^T(K\beta|\dot{\tilde{q}}|^{\beta-1}M_0^{-1}(-kM_0 sgn(S_2) + d)) \\ &\le -K\beta|\dot{\tilde{q}}|^{\beta-1}(k - ||{M_0}^{-1}d||_{max})|S_2| \le \mathbf{0} \end{aligned} \quad (26)$$

Similar to the analysis in the proof of *Theorem 2*, we can conclude that $S_2 \equiv \mathbf{0}$ for $t > t_f$. Besides, according to *Theorem 2*, we have $S_1 \equiv \mathbf{0}$ for $t \in [0, t_f]$. Consequently, the global robustness of the system is guaranteed.

Furthermore, the following nonlinear differential equation can be determined from $S_2 = \mathbf{0}$

$$\dot{\tilde{q}} = -K_1\tilde{q}^{\beta_1} \quad (27)$$

where $\beta_1 = 1/\beta$ and $K_1 = (\frac{1}{K})^{1/\beta}$. It has been described in Zak (1988, 1989) that $\tilde{q} = 0$ is the terminal attractor of the system (27). Since $\tilde{q}(t_f) = \dot{\tilde{q}}(t_f) = \mathbf{0}$, the tracking errors $\tilde{q}$ and $\dot{\tilde{q}}$ would maintain zero for $t > t_f$ by employing the control law (24).
The proof is completed.

It should be noted that the NTSMC strategy proposed in this paper doesn't utilize the finite-time convergence property. The tracking errors, which have already been driven to zero at the end of the TVSMC control process, just need to be kept at zero during the NTSMC control action.

Theorem 7. Although two different control strategies are used, the control effort is continuous at t_f as long as $n > 2$ is satisfied.

Proof. According to *Remark 4*, the TVSMC control effort at t_f can be expressed by (20). If $n > 2$, the associated control effort can be simplified as

$$u = M_0\ddot{q}_d + C_0\dot{q} + G_0 \quad (28)$$

On the other hand, the NTSMC control input is presented in (24). Since $\dot{\tilde{q}} = \mathbf{0}$ at $t = t_f$ and $1 < \beta < 2$, we can conclude that $\frac{\dot{\tilde{q}}}{K\beta|\dot{\tilde{q}}|^{\beta-1}} = \mathbf{0}$. Consequently, (24) can be described as

$$u = M_0\ddot{q}_d + C_0\dot{q} + G_0 \quad (29)$$

From (28) and (29), it can be easily drawn that the control effort is continuous at t_f.
The proof is completed.

Since the system states are kept on the sliding surface for the whole control action, global chattering problem would be experienced. To solve this problem, the continuous approximation technique is utilized in this paper. Consider the following saturation function for example,

$$sat(S) = \frac{S}{|S| + \epsilon} \quad (30)$$

where ϵ is the boundary layer thickness. The chattering will be suppressed if the boundary layer is set enough thick. However, the static error would be larger with a thicker boundary layer. As a consequence, there is a trade-off in the selection of ϵ.

To sum up, the sliding mode function for the robotic manipulators is designed as the following truncated function:

$$S = \begin{cases} (t_f - t)\dot{\tilde{q}} + n(1 - \frac{1}{e^{\tilde{q}}}) + \alpha(t) & 0 \le t \le t_f \\ \tilde{q} + K|\dot{\tilde{q}}|^{\beta} sgn(\dot{\tilde{q}}) & t > t_f \end{cases} \quad (31)$$

Correspondingly, the control effort is constructed as

$$u = \begin{cases} \frac{M_0}{t_f - t}E + C_0\dot{q} + G_0 - \upsilon M_0 sat(S) & 0 \le t \le t_f \\ M_0 F + C_0\dot{q} + G_0 - \upsilon M_0 sat(S) & t > t_f \end{cases} \quad (32)$$

where $\upsilon > ||M_0^{-1}d||_{max}$, $E = \dot{\tilde{q}} + (t_f - t)\ddot{q}_d - \frac{n\dot{\tilde{q}}}{e^{\tilde{q}}} - \begin{cases} 2At + B & 0 \le t \le T \\ 0 & t > T \end{cases}$ and $F = \ddot{q}_d - \frac{\dot{\tilde{q}}}{K\beta|\dot{\tilde{q}}|^{\beta-1}}$.

4. NUMERICAL SIMULATION

In this section, the performance of the proposed controller is demonstrated by taking the two-link rigid robotic manipulator as an example (Cong et al. (2012)). The dynamic equation of the manipulator model is given in (1) with $r = 2$, where $q = \begin{bmatrix} \theta \\ \phi \end{bmatrix}$, $M = \begin{bmatrix} M_{11}(\phi) & M_{12}(\phi) \\ M_{12}(\phi) & M_{22}(\phi) \end{bmatrix}$, $G = \begin{bmatrix} g_1(\theta,\phi)\mathrm{g} \\ g_2(\theta,\phi)\mathrm{g} \end{bmatrix}$, $C = \begin{bmatrix} -C_{12}(\phi)(\dot{\phi}) & -C_{12}(\phi)(\dot{\theta} + \dot{\phi}) \\ C_{12}(\phi)\dot{\theta} & 0 \end{bmatrix}$, $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, $u_d = \begin{bmatrix} u_{d1} \\ u_{d2} \end{bmatrix}$ with

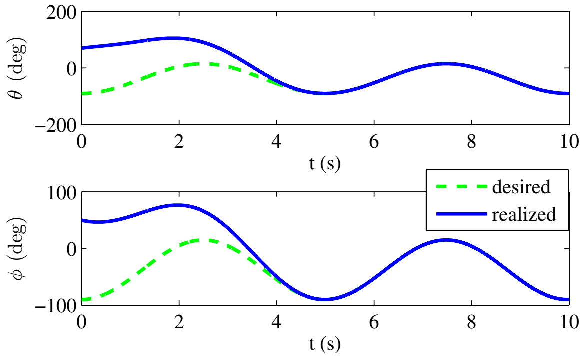

Fig. 1. Angle displacement responses

$$\begin{cases} M_{11}(\phi) = (m_1 + m_2)r_1^2 + m_2 r_2^2 + 2m_2 r_1 r_2 \cos(\phi) \\ M_{12}(\phi) = m_2 r_2^2 + m_2 r_1 r_2 \cos(\phi) \\ M_{22}(\phi) = m_2 r_2^2 \\ C_{12}(\phi) = m_2 r_1 r_2 \sin(\phi) \\ g_1(\theta, \phi) = (m_1 + m_2) r_1 \cos(\phi) + m_2 r_2 \cos(\theta + \phi) \\ g_2(\theta, \phi) = m_2 r_2 \cos(\theta + \phi) \end{cases}$$

The model parameters are selected as $l_1 = 1m$, $l_2 = 0.8m$, $m_1 = 0.5kg$ and $m_2 = 0.8kg$. The parametric uncertainties of m_1 and m_2 are assumed to be $\delta m_1 = -0.2kg$ and $\delta m_2 = -0.3kg$. Thus, the nominal values of m_1 and m_2 are $\hat{m}_1 = 0.3kg$ and $\hat{m}_2 = 0.5kg$. Furthermore, external disturbances are given as follows

$$u_d = \begin{bmatrix} 0.3sin(t + \pi/6) \\ 0.2sin(t + \pi/3) \end{bmatrix}$$

The initial angle displacements and angular velocities are $\theta(0) = 70deg$, $\phi(0) = 50deg$, $\dot{\theta}(0) = 20deg/s$ and $\dot{\phi}(0) = -20deg/s$. The reference signals are given by $\theta_d = \phi_d = -\pi/2 + 0.92(1 - cos(1.26 - t))$. Moreover, the parameters of the controller are selected as follows: $n = 3$, $T = 1$, $\beta = 5/3$, $K = 10$, $\epsilon = 1e-3$ and $\upsilon = 5$.

The trajectory tracking processes are depicted in Fig.1-Fig.4. The desired convergence time t_f is chosen as $5s$. Fig.1 shows the output tracking of the angle displacements, Fig.2 presents the tracking process of the angular velocities, Fig.3 shows the corresponding control signals and Fig.4 depicts the associated sliding mode manifold responds. It can be easily observed from Fig.1 and Fig.2 that the system states are able to track the desired reference trajectories and the tracking errors converge to zero at the given finite time t_f. Note, from Fig.3, that the control efforts are smooth and neither singularity nor chattering is experienced during the control action. Such benefits are due to the employment of the boundary layer technique and the choice of $n > 2$. As can be seen in Fig.4, the sliding mode manifolds are kept inside the boundary layer throughout the control action which implies the global sliding mode turns out to be the global sliding layer and thus, the existence of tracking errors is inevitable. However, the simulation results show that high tracking accuracy can be guaranteed with a small magnitude of boundary layer thickness.

For further analysis, the performance of the proposed control strategy is testified with respect to different n.

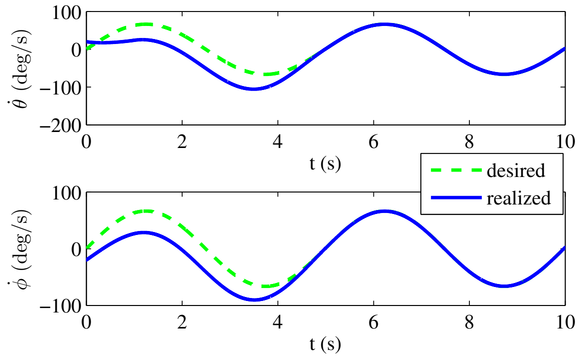

Fig. 2. Angular velocity responses

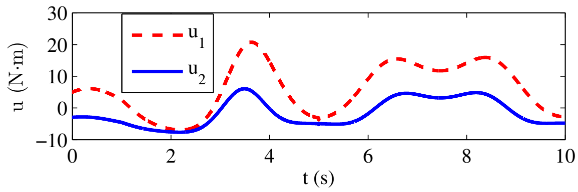

Fig. 3. Control torque responses

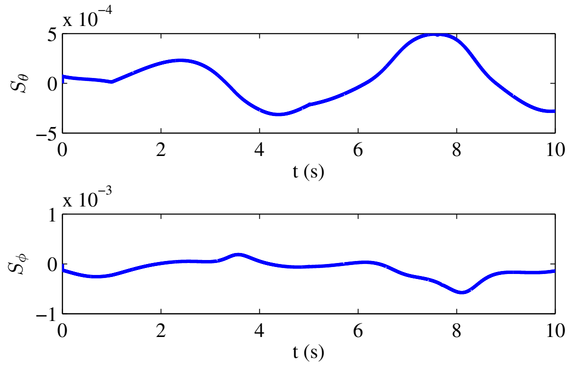

Fig. 4. Sliding mode manifold responses

The values of n is set to be 3, 4 and 5 respectively. The associated simulation results are depicted in Fig.5 - Fig.7. From this set of simulations, we can see that a larger value of n would lead to a faster convergence rate which coincides with the statement in *Remark 5*. Further, from Fig.7, it is observed that the generated control torques appear different for $t \in [0, t_f]$, however, they perform the same behavior when $t > t_f$. Such phenomena can be explained as follows. From (32), it can be found that the expression of the control efforts for $t \in [0, t_f]$ varies with n which would lead to different control torque responses. Additionally, since the expression of the control commands at $t = t_f$ is determined by (28), and q and $\dot{q}$ are equal to their desired values at that instant, the associated control torque magnitudes with respect to different n are same with each other. Consider the control efforts are not affected by n from t_f onwards, they will possess same trajectories.

5. CONCLUSION

A new approach to solve the finite-time control problem has been addressed in this paper. A key feature of the proposed method is that the tracking errors can be enforced

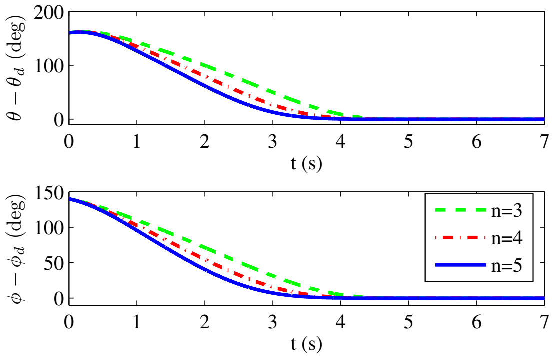

Fig. 5. Angle displacement error responses with different n

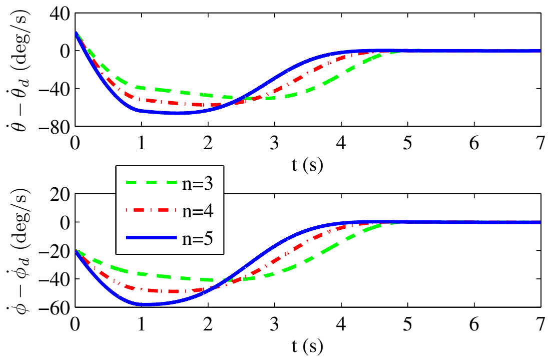

Fig. 6. Angle velocity error responses with different n

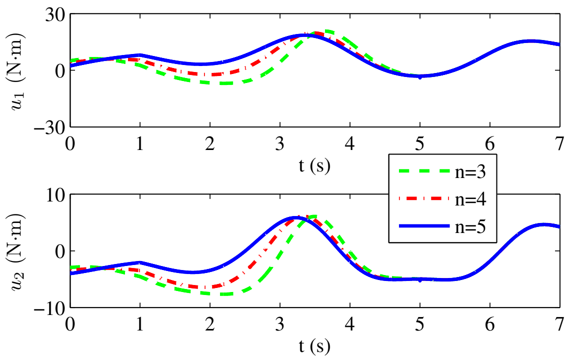

Fig. 7. Control torque responses with different n

to zero at a desired finite time in the presence of parameter variation and external disturbance. Additionally, the convergence rate can be tuned by adjusting the parameter n appropriately. The proposed controller possesses global robustness and the initial system states are free from constraints. The finite-time control strategy has been used for the control design of a rigid robotic manipulator and its effectiveness has been validated by numerical simulations. The future work could focus on extending the finite-time control strategy to high-order system category.

REFERENCES

Bartoszewicz, A. (1995). A comment on 'a time-varying sliding surface for fast and robust tracking control of second-order uncertain systems'. *Automatica*, 31 (12), 1893–1895.

Bartoszewicz, A. (1996). Time-varying sliding modes for second-order systems. *IEE Proceedings of Control Theory Application*, 143 (5), 455–462.

Berghuis, H., Ortega, R., and H.Nijmeijer (1993). A robust adaptive robot controller. *IEEE Transactions on Robotics and Automation*, 9 (6), 825–830.

Cong, B., Liu, X., and Chen, Z. (2012). Disturbance observer based time-varying sliding mode control for uncertain mechanical system. *Journal of Systems Engineering and Electronics*, 23 (1), 108–117.

Du, H., Li, S., and Qian, C. (2011). Finite-time attitude tracking control of spacecraft with application to attitude synchronization. *IEEE Transactions on Automatic Control*, 56 (11), 2711–2717.

Feng, Y., Yu, X., and Man, Z. (2002). Non-singular terminal sliding mode control of rigid manipulators. *Automatica*, 38, 2159–2167.

Khan, M.F., ul Islam, R., and Iqbal, J. (2012). Control strategies for robotic manipulators. In *IEEE International Conference on Robotics and Artificial Intelligence (ICRAI)*, 26–33.

Laghrouche, S., Plestan, F., and Glumineau, A. (2007). A higher order sliding mode control based on integral sliding mode. *Automatica*, 43, 531–537.

Li, T.H. and Huang, Y.C. (2010). MIMO adaptive fuzzy terminal sliding-mode controller for robotic manipulators. *Information Sciences*, 180, 4641–4660.

Neila, M. and Tarak, D. (2011). Adaptive terminal sliding mode control for rigid robotic manipulators. *International Journal of Automation and Computing*, 8 (2), 215–220.

Ortega, R. and Spong, M. (1989). Adaptive motion control of rigid robots: a tutorial. *Automatica*, 25, 877–888.

Slotine, J. and Li, W. (1989). Composite adaptive control of robot manipulator. *Automatica*, 25, 509–519.

Slotine, J. and Li, W. (1991). *Applied nonlinear control.* Prentice Hall, New Jersey.

Wang, X., Sun, X., and Ye, H. (2012). Finite-time position tracking control of rigid hydraulic manipulators based on high-order terminal sliding mode. In *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of System and Control Engineering*, 394–415.

Wang, Y., Wang, C., Lu, P., and Wang, Y. (2011). High-order nonsingular terminal sliding mode optimal control of two-link flexible manipulators. In *IEEE 37th Annual Conference on IEEE Industrial Eletronics Society (S-TACS)*, 3953–3958.

Wu, Y. and Yu, X. (1998). Terminal sliding mode control design for uncertain dynamic systems. *System & Control Letters*, 34, 281–287.

Yu, S., Yu, X., Shirinzadeh, B., and Man, Z. (2005). Continuous finite-time control for robotic manipulators with terminal sliding mode. *Automatica*, 41, 1957–1964.

Yu, X. and Man, Z. (2002). Fast terminal sliding-mode control design for nonlinear dynamical systems. *IEEE Transactions on Circuit and Systems*, 49 (2), 261–264.

Zak, M. (1988). Terminal atractors for addressable memory in neural network. *Physics Letters*, 133, 218–222.

Zak, M. (1989). Terminal atractors in neural network. *Neural Networks*, 2, 259–274.