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Hybrid Terrestrial and Aerial Quadrotor Control

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Abstract: A recent breakthrough in aerial mobile robotics appears with the quadrotor platform. It is particularly suitable for indoor exploration because of its high stability, manoeuvrability and because of the possibility to hover. However quadrotors suffer from their energy consumption and can only fly about 15 minutes. A new design of quadrotor is then proposed in this paper and allows it to move on the ground for energy saving purposes. This paper only focuses on the terrestrial control. A 3DOF non linear dynamic modeling of the terrestrial displacement is firstly obtained and then control laws for trajectory tracking and point stabilization are designed. A flatness approach is used for tracking purpose and a non linear time-varying control law is proposed for point stabilization. The second approach is justified by the fact that the system does not satisfy Brockett's theorem. Simulation results also underline the similar dynamic behaviour of our system with hovercrafts.

1. INTRODUCTION

In the context of indoor exploration with mobile robots, quadrirotors could be a good solution: they are suitable to avoid obstacles such stairs, steps, desks... However the autonomy in terms of energy is the main drawback of such a system with usually 15 minutes of fly. In this perspective, we propose a study on a new design of a quadrirotor which allows it to move on the ground when flying is not necessary. As far as is our knowledge, two "Hybrid Terrestrial and Aerial Quadrotor" system exists : HytaQ [2012] and BFly [2013] ; they can be used for both indoor and outdoor environment. Their design allows the drone to extend the autonomy for longer operations. Our design is quite different and can be only used on flat indoor surfaces.

This paper will only focus on the 3DOF non linear terrestrial modeling and control of our hybrid system. Flying controls are quite known and standard for a quadrirotor, but in our case we will only investigate the terrestrial control, and propose control laws to track arbitrary terrestrial trajectories on a xy-plane.

It will be pointed out that the terrestrial model for our drone has similarities with models of hovercraft. Besides models of hovercraft are also similar to marine vehicles ones such as surface vessels for example. Indeed a model is suggested in Fantoni et al. [1999] and is directly derived from a simplifed ship model. It gathers kinematic and dynamic equations. The model proposed in the present paper is slightly different: first because it relies only on the dynamic equations and secondly because of the control inputs of our system (section II). Different control laws have been proposed in the litterature for this kind of underactuated vehicles. For a large class of underactuated vehicles in which our terrestrial quadrotor is included, it is shown in Pettersen et al. [1996] that they cannot be asymptotically stabilized by continuous state feedback for point stabilization at the origin ; they do not satisfy Brockett's theorem (Brockett [1983]). Thus the control laws can be divided into two subsets : one for trajectory tracking purposes and the other for point stabilization.

To tackle the first issue, Godhavn [1996] presents a control law based on backstepping providing exponential tracking of time-dependent smooth trajectories made by straight lines or arcs of circles. These trajectories aim to be feasible and consistent with the vehicles' dynamics. Moreover the control is obtained from models of underactuated surface vessels and is only valid for a surge velocity different from zero. Aguiar et al. [2003] considers a position tracking problem rather than "state space" feasible trajectories. Contrary to Godhavn [1996], Aguiar et al. [2003] focuses on specific hovercraft systems. The non linear control law used there is based on a Lyapunov approach that exponentially stabilizes the position tracking error to a neighborhood of the origin. This controller was validated experimentally on a platform (the Caltech Multi-Vehicle Wireless Testbed -MVWT-) almost similar to ours. Other approaches were investigated for example in Sira-Ramírez et al. [2000] where the flatness property of the hovercraft system is explored and a flat control law is then proposed. However the reference trajectories to track have to avoid some singularities in the control. Sira-Ramírez [2002] started from this previous results and developed a robust control for hovercraft by the use of a dynamic second order sliding mode control.

Finally, a lot of solutions were proposed to overcome the trajectory tracking issues but without taking into account the point stabilization challenge. The problem is investigated in Pettersen et al. [1996], Pettersen et al [1997] and Pettersen et al. [1998] where a time varying control law for stabilizing an equilibrium is used. Experiments were realized and presented in Pettersen et al. [1998]. Fantoni

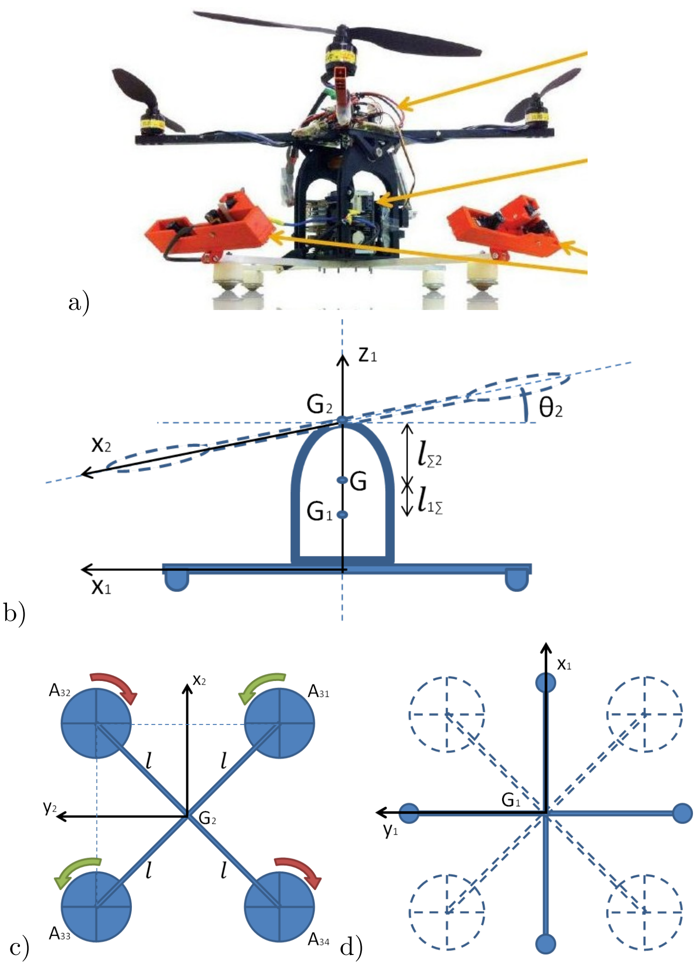

Fig. 1. Hybrid quadrotor structure: a) Picture of the platform. b) Side view of solids S_1 and $S_2 \cup S_{3i}$ respectively in solid and dotted lines. c) Top view of $S_2 \cup S_{3i}$. d) Top view of solids S_1 and $S_2 \cup S_{3i}$ respectively in solid and dotted lines.

et al. [1999] uses a Lyapunov approach for the convergence analysis of discontinuous controllers.

The paper is organized as follows: in section II a 3DOF modeling of the terrestrial displacement dynamics is given. Then section III presents two control laws, one for the tracking problem using the flatness property of our system and the other one for the point stabilization concern, using a time-varying approach. Finally, in section IV some simulation results are presented.

2. TERRESTRIAL QUADROTOR DYNAMIC MODEL

2.1 System description and notations

We consider two rigid bodies written S_1, S_2 of weight M_1 and M_2 and four others written S_{3j} for $j \in [1;4]$ of respective weight m_{3j}. The body S_1 represents the base of the drone (figure 1-d) where four omnidirectional ball casters are fixed and allow the drone to move freely in all directions even sideways. It supports the body S_2 (figure 1-b, 1-c, 1-d) through a revolute joint at the point G_2 and can be moved by a servo-motor of torque Γ_{servo}. The four other bodies are the propellers of the drone. The center of mass of these bodies are respectively $G_1, G_2, A_{31}, A_{32}, A_{33}$ and A_{34}. The center of mass of the whole system Σ of weight M is G. The frames corresponding to the bodies are denoted $R_1(\overrightarrow{x_1}, \overrightarrow{y_1}, \overrightarrow{z_1}), R_2(\overrightarrow{x_2}, \overrightarrow{y_2}, \overrightarrow{z_2}), R_{3i}(\overrightarrow{x_{3i}}, \overrightarrow{y_{3i}}, \overrightarrow{z_{3i}})$ for $i \in [1,4]$ and R_0 for the inertial frame. We can also denote the position $\overrightarrow{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{R_0}$, the velocity $\overrightarrow{V} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}_{R_1} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{R_0}$ and angular velocities $\overrightarrow{\Omega_{R_1/R_0}} = \begin{pmatrix} p_1 \\ q_1 \\ r_1 \end{pmatrix}_{R_1} = \begin{pmatrix} 0 \\ 0 \\ r_1 \end{pmatrix}_{R_1}$, $\overrightarrow{\Omega_{R_2/R_1}} = \begin{pmatrix} p_2 \\ q_2 \\ r_2 \end{pmatrix}_{R_2} = \begin{pmatrix} 0 \\ q_2 \\ 0 \end{pmatrix}_{R_2}$ and finally $\overrightarrow{\Omega_{R_{3i}/R_2}} = \begin{pmatrix} 0 \\ 0 \\ \omega_i \end{pmatrix}_{R_{3i}}$ for i from 1 to 4. The orientation of all the bodies can be defined considering two Euler angles: the yaw Ψ_1 for S_1 and the pitch θ_2 for S_2 such that refering to Bertrand [2007] we can write:

$$[u]_{R_0} = R_{Z_{R_1}}(\psi_1)^\top [u]_{R_1} \tag{1}$$

$$[u]_{R_1} = R_{Y_{R_2}}(\theta_2)^\top [u]_{R_2} \tag{2}$$

with

$$R_Z(\psi_1) = \begin{pmatrix} C_{\psi_1} & S_{\psi_1} & 0 \\ -S_{\psi_1} & C_{\psi_1} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_Y(\theta_2) = \begin{pmatrix} C_{\theta_2} & 0 & -S_{\theta_2} \\ 0 & 1 & 0 \\ S_{\theta_2} & 0 & C_{\theta_2} \end{pmatrix}$$

Besides in this particular case these Euler angles can be linked to angular velocities in this way:

$$\begin{cases} \dot{\theta}_2 = q_2 \\ \dot{\Psi}_1 = r_1 \end{cases} \tag{3}$$

2.2 Newton's second law of motion

It can be written:

$$M\dot{\overrightarrow{V_G}} = M\overrightarrow{g} + \sum_{i=1}^{4} \overrightarrow{F_i} \tag{4}$$

$$\overrightarrow{\delta_G^{\Sigma}}.\overrightarrow{z_1} = \sum_{i=1}^{4} (\overrightarrow{GA_{3i}} \times \overrightarrow{F_i}).\overrightarrow{z_1} + \overrightarrow{M_i}.\overrightarrow{z_1} \tag{5}$$

$$\overrightarrow{\delta}_G^{S_2 \cup S_{3i}}.\overrightarrow{y_1} = \sum_{i=1}^{4} (\overrightarrow{G_2A_{3i}} \times \overrightarrow{F_i}).\overrightarrow{y_1} + \Gamma_{servo} \tag{6}$$

$$\overrightarrow{\delta_{A_{3i}}^{S_{3i}}}.\overrightarrow{z_2} = \overrightarrow{M_i}.\overrightarrow{z_2} + \varepsilon_i \Gamma_i \tag{7}$$

with $\times$ the cross product, F_i and M_i the aerodynamic forces and momentum generated by the propellers applied at A_{3i}. They can be respectively expressed as follows: $\overrightarrow{F_i} = a\omega_i^2 \overrightarrow{z_2}$ and $\overrightarrow{M_i} = -b\varepsilon_i \omega_i^2 \overrightarrow{z_2}$ where $\varepsilon_i = (-1)^{i+1}$ (Martin et al. [2009]). Γ_i is the torque of the brushless motor at point A_{3i}. $\overrightarrow{\delta_G^{\Sigma}}$, $\overrightarrow{\delta_{G_2}^{S_2}}$ and $\overrightarrow{\delta_{A_{3i}}^{S_{3i}}}$ are the angular momentum of the rigid bodies Σ, S_2 and S_{3i}. One assumption which will be supposed in the remainder of this paper is that points G, G_1 and G_2 are considered identical.

Acceleration calculation

$$M\begin{pmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{pmatrix}_{R_0} = M\begin{pmatrix}0\\0\\-g\end{pmatrix}_{R_0} + R_{Z_{R_1}}(\psi_1)^\top R_{Y_{R_2}}(\theta_2)^\top \begin{pmatrix}0\\0\\ a\sum_{i=1}^{4}\omega_i^2\end{pmatrix}_{R_2} \tag{8}$$

This finally leads to:

$$M\begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix} = \begin{pmatrix} aC_{\psi_1}S_{\theta_2}\sum_{i=1}^{4}\omega_i^2\\ aS_{\psi_1}S_{\theta_2}\sum_{i=1}^{4}\omega_i^2\end{pmatrix} \tag{9}$$

Dynamic momentum of Σ Considering the center of mass G, the dynamic momentum can be associated with the kinematic momentum as follows: $\overrightarrow{\sigma_G} = \overrightarrow{\delta_G}$. Moreover $\overrightarrow{\sigma_G^{\Sigma}} = \overrightarrow{\sigma_G^{S_1}} + \overrightarrow{\sigma_G}^{S_2\cup S_{3i}}$, then the dynamic momentum can be expressed as follows:

$$\overrightarrow{\delta_G^{\Sigma}} = \left[\frac{d\overrightarrow{\sigma_G^{\Sigma}}}{dt}\right]_{R_0} = \left[\frac{d\overrightarrow{\sigma_G^{S_1}}}{dt}\right]_{R_0} + \left[\frac{d\overrightarrow{\sigma_G}^{S_2\cup S_{3i}}}{dt}\right]_{R_0} \tag{10}$$

2.2.2.1. Kinematic momentum of S_1 We can recall that the assumption $G = G_1$ simplifies the equations:

$$\overrightarrow{\sigma_G^{S_1}} = J_G(S_1)\overrightarrow{\Omega}(R_1/R_0) \tag{11}$$

$$\left[\frac{d\overrightarrow{\sigma_G^{S_1}}}{dt}\right]_{R_0} = J_G(S_1)\left[\frac{d\overrightarrow{\Omega}(R_1/R_0)}{dt}\right]_{R_1} + \overrightarrow{\Omega}(R_1/R_0)\times J_G(S_1)\overrightarrow{\Omega}(R_1/R_0)$$
$$= \begin{bmatrix} I_1\dot{p_1}\\ I_1\dot{q_1}\\ J_1\dot{r_1}\end{bmatrix}_{R_1} + \begin{bmatrix}(J_1-I_1)r_1q_1\\ (I_1-J_1)r_1p_1\\ 0\end{bmatrix}_{R_1} = \begin{bmatrix}0\\0\\J_1\dot{r_1}\end{bmatrix}_{R_1} \tag{12}$$

2.2.2.2. Kinematic momentum of $S_2 \cup S_{3i}$ It is common (Martin et al. [2009]) to approximate $S_2 \cup S_{3i}$ by a rigid body $S_{2''}$ which is composed by S_2 and four disks representing the propellers with same masses and radii for the inertia calculation. The assumption is valid because propellers have a much faster dynamics than the system Σ considered. The following equation is then proved in Martin et al. [2009]:

$$\overrightarrow{\sigma_G}^{S_2\cup S_{3i}} = J_G^{S_{2''}}\overrightarrow{\Omega}(R_2/R_0) + \sum_{i=1}^{4}\varepsilon_i\omega_i\left(J_{A_{3i}}^{S_{3i}}\overrightarrow{z_2}\right) \tag{13}$$

with

$$J_{A_{3i}}^{S_{3i}} = \begin{pmatrix}0&0&0\\0&0&0\\0&0&J_3\end{pmatrix}_{R_{3i}}$$

$$(J_G^{S_{2''}})_{R_1} = R_{Y_{R_2}}(\theta_2)\top\begin{pmatrix}I_{2''}&0&0\\0&I_{2''}&0\\0&0&J_{2''}\end{pmatrix}_{R_2} R_{Y_{R_2}}(\theta_2)$$

Using $\overrightarrow{\Omega}(R_2/R_0) = \overrightarrow{\Omega}(R_2/R_1) + \overrightarrow{\Omega}(R_1/R_0) = \begin{bmatrix}0\\q_2\\r_1\end{bmatrix}_{R_1}$, and the derivative in the inertial frame of equation (13), we obtain:

$$\overrightarrow{\delta_G^{S_2\cup S_{3i}}}.\overrightarrow{y_1} = I_{2''}\dot{q_2} + B(\theta_2)r_1^2 + r_1\sum_{i=1}^{4}\varepsilon_i\omega_i J_3\sin(\theta_2) \tag{14}$$

and

$$\overrightarrow{\delta_G^{S_2\cup S_{3i}}}.\overrightarrow{z_1} = \dot{r_1}C(\theta_2) - r_1q_2\sin(2\theta_2)(J_{2''}-I_{2''}) + J_3\cos(\theta_2)\sum_{i=1}^{4}\varepsilon_i\dot{\omega}_i - q_2J_3\sin(\theta_2)\sum_{i=1}^{4}\varepsilon_i\omega_i \tag{15}$$

with $B(\theta_2) = \cos(\theta_2)\sin(\theta_2)(J_{2''} - I_{2''})$ and $C(\theta_2) = \frac{1}{2}\left((I_{2''}+J_{2''}) + \cos(2\theta_2)(J_{2''}-I_{2''})\right)$.

2.2.2.3. External momentum applied to the system Σ Let us consider the right part of equation (5) and respecting the notations of Fig. 1, we have:

$$\sum_{i=1}^{4}\overrightarrow{GA_{3i}}\times\overrightarrow{F_i} = \begin{bmatrix} a\frac{l}{\sqrt{2}}(-\omega_1^2+\omega_2^2+\omega_3^2-\omega_4^2)\\ a\frac{l}{\sqrt{2}}(-\omega_1^2-\omega_2^2+\omega_3^2+\omega_4^2)\\ 0\end{bmatrix}_{R_2}$$
$$= \begin{bmatrix}\cos(\theta_2)\frac{al}{\sqrt{2}}(-\omega_1^2+\omega_2^2+\omega_3^2-\omega_4^2)\\ \frac{al}{\sqrt{2}}(-\omega_1^2-\omega_2^2+\omega_3^2+\omega_4^2)\\ -\sin(\theta_2)\frac{al}{\sqrt{2}}(-\omega_1^2+\omega_2^2+\omega_3^2-\omega_4^2)\end{bmatrix}_{R_1} \tag{16}$$

$$\overrightarrow{M_i} = \begin{bmatrix}-b\varepsilon_i\omega_i^2\sin(\theta_2)\\0\\-b\varepsilon_i\omega_i^2\cos(\theta_2)\end{bmatrix}_{R_1} \tag{17}$$

2.3 Complete dynamic model

Finally, if one summarizes all equations (4), (5), (6), (9), (12), (14), (15), (16), (17) and if one neglects gyroscopic effects of the propellers because the inertia J_{3i} is much smaller than the total inertia, one obtains the following system:

$$\begin{cases}\ddot{x} = C_{\psi_1}S_{\theta_2}u_1\\ \ddot{y} = S_{\psi_1}S_{\theta_2}u_1\\ \ddot{\psi_1} = \frac{S_{\theta_2}}{C(\theta_2)+J_1}u_2 + \frac{C_{\theta_2}}{C(\theta_2)+J_1}u_3 + \dot{\psi_1}\dot{\theta_2}\frac{\sin(2\theta_2)(J_{2''}-I_{2''})}{C(\theta_2)+J_1}\\ \ddot{\theta_2} = u_4 - \frac{B(\theta_2)}{I_{2''}}\dot{\psi_1}^2\end{cases} \tag{18}$$

with four independent controls:

$$\begin{cases} u_1 = \frac{a}{M}\sum_{i=1}^{4}\omega_i^2\\ u_2 = \frac{al}{\sqrt{2}}(\omega_1^2-\omega_2^2-\omega_3^2+\omega_4^2)\\ u_3 = -b(\omega_1^2-\omega_2^2+\omega_3^2-\omega_4^2)\\ u_4 = \frac{al}{\sqrt{2}I_{2''}}(\omega_3^2+\omega_4^2-\omega_1^2-\omega_2^2) + \frac{\Gamma_{servo}}{I_{2''}}\end{cases} \tag{19}$$

It can be pointed out that u_1 is always positive which means that the propellers cannot provide forces in the opposite direction but this can be realized using the variable θ_2 (see equation (18)). In the next part of this paper, we will not consider the control of the propellers (7) because their dynamics are much faster than the dynamics of the whole system and could be taken into account using a singular perturbation approach (see H.K. Khalil [1995]).

3. THE CONTROL APPROACH

The equations of the dynamic model (18) are similar to the equations of the model for hovercraft described for example in Sira-Ramírez et al. [2000]. It could have been predicted regarding the physical design of the terrestrial drone but it is especially in the simulation results (Fig. 2_b) that it is revealed. Indeed the orientation of the drone symbolized by the arrow in the mentioned graph shows that it can move sideway and that the yaw angle is still in the direction of the acceleration vector. Besides this last remark is also justified in the flatness study (23). In this section we will detail the flatness control used for tracking trajectory purposes and we will follow by a time-varying approach for point stabilization. The system of equations can be rewritten in this way:

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \dot{\psi}_1 \\ \ddot{\theta}_2 \end{pmatrix} = F \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} + G \tag{20}$$

with

$$F = \begin{pmatrix} \cos(\psi_1)\sin(\theta_2) & 0 & 0 & 0 \\ \sin(\psi_1)\sin(\theta_2) & 0 & 0 & 0 \\ 0 & \dfrac{\sin(\theta_2)}{C(\theta_2)+J_1} & \dfrac{\cos(\theta_2)}{C(\theta_2)+J_1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{21}$$

It can be noticed that a static feedback is not possible because $det(F) = 0$; that is why we explored a dynamic feedback approach.

3.1 Tracking trajectory: a flat control law

We proceeded by using a dynamic extension delaying the action of the control u_1, we introduced a new state η so that $\dot{\eta} = v_1$ (22). Moreover it appears that u_2 and u_3 both act on the dynamics of ψ_1. These two controls can be expressed as a function of a new one called for example u' so that $u_2 = \sin(\theta_2)(C(\theta_2) + J_1)u'$ and $u_3 = \cos(\theta_2)(C(\theta_2) + J_1)u'$. However we focused our study only on the cascade system below:

$$\begin{cases} \ddot{x} = \cos(\psi_1)\sin(\theta_2)\eta \\ \ddot{y} = \sin(\psi_1)\sin(\theta_2)\eta \\ \dot{\eta} = v_1 \\ \dot{\psi}_1 = v_2 \\ \dot{\theta}_2 = v_3 \end{cases} \tag{22}$$

with v_1, v_2 and v_3 being the new control inputs. Backstepping methods are then used to recover the original control inputs described in (18) (see for example Sepulchre et al. [1992]). In order to develop a flatness approach, it is necessary to identify the flat outputs of the system. We recall the definition of a flat system (see e.g. Fliess et al. [1995]).

Definition The system defined by $\dot{x} = f(x, u)$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, is said to be flat if there exist a function $h : \mathbb{R}^n \times (\mathbb{R}^m)^{r+1} \rightarrow \mathbb{R}^m$, a function $\phi : (\mathbb{R}^m)^r \rightarrow \mathbb{R}^n$ and $\psi : (\mathbb{R}^m)^{r+1} \rightarrow \mathbb{R}^m$ such that:

$$y = h(x, u, \dot{u}, ..., u^{(r)})$$
$$x = \phi(y, \dot{y}, ..., y^{(r-1)})$$
$$u = \psi(y, \dot{y}, ..., y^{(r-1)}, y^{(r)})$$

y is called the flat output of the system.

Proposition 1. x, y and θ_2 are flat outputs of system (22).

Proof. The whole state of system (22) can be expressed as a function of x, y and θ_2:

$$\begin{cases} x = x \\ y = y \\ \eta = \sqrt{\dfrac{\ddot{x}^2 + \ddot{y}^2}{\sin^2(\theta_2)}} \\ \psi_1 = \arctan(\dfrac{\ddot{y}}{\ddot{x}}) \\ \theta_2 = \theta_2 \end{cases} \tag{23}$$

In order to linearize the system, the first two equations of (22) are time-differentiated to make the controls appear.

$$\begin{pmatrix} \dddot{x} \\ \dddot{y} \\ \dot{\theta}_2 \end{pmatrix} = \begin{pmatrix} \cos(\psi_1)\sin(\theta_2) & -\eta\sin(\psi_1)\sin(\theta_2) & \eta\cos(\psi_1)\cos(\theta_2) \\ \sin(\psi_1)\sin(\theta_2) & \eta\cos(\psi_1)\sin(\theta_2) & \eta\sin(\psi_1)\cos(\theta_2) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \tag{24}$$

The determinant of the above matrix is $\eta\sin^2(\theta_2)$. The flat control law is then only valid for trajectories where $\eta \neq 0$ and $\theta_2 \neq 0$ and we have:

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} \dfrac{\cos(\psi_1)}{\sin(\theta_2)} & \dfrac{\sin(\psi_1)}{\sin(\theta_2)} & -\eta\dfrac{\cos(\theta_2)}{\sin(\theta_2)} \\ -\dfrac{\sin(\psi_1)}{\eta\sin(\theta_2)} & \dfrac{\cos(\psi_1)}{\eta\sin(\theta_2)} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \tag{25}$$

where V_1, V_2, V_3 are the new auxiliary inputs and this leads to:

$$\begin{aligned} v_1 &= \frac{\cos(\arctan(\frac{\ddot{y}}{\ddot{x}}))}{\sin(\theta_2)} V_1 + \frac{\sin(\arctan(\frac{\ddot{y}}{\ddot{x}}))}{\sin(\theta_2)} V_2 - \sqrt{\ddot{x}^2 + \ddot{y}^2}\frac{\cos(\theta_2)}{\sin^2(\theta_2)} V_3 \\ v_2 &= -\frac{\sin(\arctan(\frac{\ddot{y}}{\ddot{x}}))}{\sqrt{\ddot{x}^2 + \ddot{y}^2}} V_1 + \frac{\cos(\arctan(\frac{\ddot{y}}{\ddot{x}}))}{\sqrt{\ddot{x}^2 + \ddot{y}^2}} V_2 \\ v_3 &= V_3 \end{aligned} \tag{26}$$

with for example

$$\begin{aligned} V_1 &= \dddot{x}_{ref} + k_{2x}(\ddot{x}_{ref} - \ddot{x}) + k_{1x}(\dot{x}_{ref} - \dot{x}) \\ &+ k_{0x}(x_{ref} - x) + k_{-1x}(\int x_{ref}dt - \int xdt) \end{aligned} \tag{27}$$

V_2 and V_3 are expressed in a similar way with gains chosen to satisfy the Routh-Hurwitz stability criterion. This control law globally exponentially stabilizes the tracking errors while respecting $\eta_{ref} \neq 0$ and $\theta_{2_{ref}} \neq 0$. This is then a dynamic feedback solution to trajectory tracking issue based on flatness and exact tracking error linearization. We will now study the point stabilisation problem where a continuous state feedback is not possible because the system does not satisfy Brockett's theorem (Brockett [1983]). We refer to Coron et al. [1992] but also to Coron [1992], Samson [1992] or Samson [1995].

3.2 Point stabilization: a time-varying control law

One objective for the control of the terrestrial drone is to be able to stop it at a point $(\overline{x}, \overline{y})$ that is to say at the origin of the system (28):

$$\begin{cases} \ddot{x} = \cos(\psi_1)p \\ \ddot{y} = \sin(\psi_1)p \\ p = S_\theta u_1 \\ \dot{\psi}_1 = v_2 \\ \dot{\theta}_2 = v_3 \end{cases} \tag{28}$$

As stated earlier, this system of equations cannot have a stabilizing control law around a point by a continuous state feedback according to Brockett's theorem (Brockett [1983]). The papers Coron et al. [1992], Coron [1992], Samson [1992] and Samson [1995] proposed stabilizing control laws based on a time-varying approach. In Coron et al. [1992], it is shown how to build such laws to stabilize a system of the following form $\dot{x} = vf(x, u)$ with $v = v(x, t)$ and $u = u(x, t)$. We based our work on the theory developed in Coron et al. [1992] and on the practical case presented in the same paper on an unicycle system. According to Coron et al. [1992], $u(x, t) = w(x)g(t)$ and $v(x, t) = -q(x, u(x, t))$ stabilize $\dot{x} = vf(x, u)$ to the point $x = 0$ if and only if some properties are satisfied for the case $dim(u) = 1$:

(1)

$$\begin{gathered} g \in C^{\infty}(\mathbb{R}, [-1; 1]) \\ \forall t \in \mathbb{R} \quad g(t+T) = g(t) \\ \forall t \in \mathbb{R}, \quad \exists l \in \mathbb{N} - \{0\} \quad g^{(l)}(t) \neq 0 \\ \text{for example,} \quad g(t) = \sin(\frac{2\pi t}{T}) \end{gathered}$$

(2)

$$\begin{gathered} w : \mathbb{R}^n \to \mathbb{R}, \quad w \in C^{\infty}(\mathbb{R}^n, \mathbb{R}) \\ w(0) = 0 \\ \forall x \neq 0, \quad w(x) = 0 \Rightarrow q(x, 0) \neq 0 \\ \begin{cases} \forall x \quad s.t. \quad w(x) \neq 0, \quad \exists r \in \mathbb{N} \quad s.t. \\ \forall u \in [-|w(x)|, |w(x)|], \quad \dfrac{\partial^r q}{\partial u^r} \neq 0 \end{cases} \end{gathered}$$

(3)

$$\begin{gathered} q : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R} \\ q(x, u) = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i}(x) f_i(x, u) \end{gathered}$$

with V a Lyapunov function such that

$$\begin{gathered} V : \mathbb{R}^n \to \mathbb{R} \\ V(x) \to +\infty \quad \text{when} \quad |x| \to +\infty \\ V(0) = 0 \\ \forall x \in \mathbb{R}^n - \{0\} \quad V(x) > 0 \end{gathered}$$

Finally if these conditions are respected, $0 \in \mathbb{R}^n$ is globally asymptotically stable for $\dot{x} = v(x, t)f(x, u(x, t))$. In order to apply this method to our system, we first simplify it by (29):

$$\begin{cases} \ddot{x} = \cos(\psi_1)p \\ \ddot{y} = \sin(\psi_1)p \end{cases} \tag{29}$$

The main idea developped in Coron et al. [1992] comes from the derivative of the Lyapunov function $V = \frac{\dot{x}}{2} + \frac{\dot{y}}{2}$. Indeed $\dot{V} = p(cos(\psi_1)\dot{x} + sin(\psi_1)\dot{y})$, this justifies the choice of $p = -(cos(\psi_1)\dot{x} + sin(\psi_1)\dot{y})$. It is also shown in the paper how to choose ψ_1 such that the system (29) converges to the origin satisfying the LaSalle theorem. A comparison with the practical case described in Coron et al. [1992] leads to:

- $u(\dot{x}, \dot{y}, t) = \psi_1(\dot{x}, \dot{y}, t)$, $v(\dot{x}, \dot{y}, t) = p(\dot{x}, \dot{y}, t)$ and $f(\dot{x}, \dot{y}, u) = (\cos(u), \sin(u))$.
- Property (3) is satisfied with $V(\dot{x}, \dot{y}) = \frac{\dot{x}^2 + \dot{y}^2}{2}$ and $q(\dot{x}, \dot{y}, u) = \dot{x}\cos(u) + \dot{y}\sin(u)$
- Properties (1) and (2) are verified using $w(\dot{x}, \dot{y}) = \dot{y}$ and $g(t) = \sin(\frac{2\pi t}{T})$

$$\begin{gathered} \forall x \neq 0, \quad w(x) = 0 \Rightarrow \dot{x} \neq 0 \Rightarrow q(\dot{x}, \dot{y}, 0) = \dot{x} \neq 0 \\ \forall x \quad s.t. \quad w(x) \neq 0 \Rightarrow \dot{y} \neq 0, \quad \exists r \in \mathbb{N} \quad s.t. \\ \forall u \in [-|w(x)|, |w(x)|], \quad \frac{\partial^r q}{\partial u^r} \neq 0 \end{gathered}$$

It can then be written:

$$\begin{gathered} \psi_1(\dot{x}, \dot{y}, t) = \dot{y}\sin(\frac{2\pi t}{T}) \\ p(\dot{x}, \dot{y}, t) = -\dot{x}\cos(\dot{y}\sin(\frac{2\pi t}{T})) - \dot{y}\sin(\dot{y}\sin(\frac{2\pi t}{T})) \end{gathered} \tag{30}$$

Cascaded system In order to stabilize the complete system (28), we have now to consider the problem of stabilizing the cascaded system (31).

$$\begin{cases} \ddot{x} = \cos(\psi_1)p \\ \ddot{y} = \sin(\psi_1)p \\ \dot{\psi}_1 = v_2 \\ \dot{p} = \overline{v} \end{cases} \tag{31}$$

Lemma 1 in Coron et al. [1992] proposes a method to solve it. It is then applied for the practical case of the unicycle model. Considering previous control laws (30), let us detail some new notations ($'$ denoting the transpose operation):

- $Y = (p, \psi_1)'$, $X = (\dot{x}, \dot{y})'$, $u_2(X, t) = (u_{21}, u_{22})'$
- $F(X, Y) = (\cos(\psi_1)p, \sin(\psi_1)p)'$
- $u_{21}(X, t) = p(\dot{x}, \dot{y}, t) = -\dot{x}\cos(\dot{y}\sin(\frac{2\pi t}{T})) - \dot{y}\sin(\dot{y}\sin(\frac{2\pi t}{T}))$
- $u_{22}(X, t) = \psi_1(\dot{x}, \dot{y}, t) = \dot{y}\sin(\frac{2\pi t}{T})$
- $V(X) = \frac{\dot{x}^2 + \dot{y}^2}{2}$

We define $\overline{u}(X, Y, t) = (\overline{v}, v_2)'$ such that:

$$\begin{gathered} \overline{u} = \frac{\partial u_2}{\partial t}(X, t) - (Y - u_2(X, t)) + \Sigma_{i=1}^{n} \frac{\partial u_2}{\partial X_i}(X, t) F_i(X, Y) \\ -H(X, u_2(X, t), Y)(\frac{\partial V}{\partial X}(X, t))' \end{gathered} \tag{32}$$

with $H \in C^{\infty}(\mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^m; L(\mathbb{R}^m; \mathbb{R}^n))$ such that:

$$F(X, Y) - F(X, u_2) = H'(X, u_2, Y)(Y - u_2) \tag{33}$$

Coron et al. [1992] proves that $\overline{u}(X, Y, t)$ globally asymptotically stabilizes (31) considering the following Lyapunov function:

$$W(X, Y, t) = \frac{1}{2}|Y - u_2(X, t)|^2 + V(X, t) \tag{34}$$

Applying this method to our system, we finally obtain control laws that stabilize (31) :

- $\overline{v}(X, Y, t) = \dot{u_{21}} - \dot{x}\cos(u_{22}) - \dot{y}\sin(u_{22}) - (p - u_{21})$
- $v_2(X, Y, t) = \dot{u_{22}} + \dot{x}p\sin(\frac{\psi_1 + u_{22}}{2})\sin_c(\frac{\psi_1 - u_{22}}{2}) - \dot{y}p\cos(\frac{\psi_1 + u_{22}}{2})\sin_c(\frac{\psi_1 - u_{22}}{2}) - (\psi_1 - u_{22})$
- $W(X, Y, t) = \frac{\dot{x}^2 + \dot{y}^2 + (\psi_1 - \dot{y}\sin(\frac{2\pi t}{T}))^2 + (p + \dot{x}\cos(\dot{y}\sin(\frac{2\pi t}{T})) + \dot{y}\sin(\dot{y}\sin(\frac{2\pi t}{T})))^2}{2}$

4. SIMULATION RESULTS

Some simulation results are shown (Fig. 2) in this section where the two control laws described in the present paper are implemented in a hybrid controller. In the scenario presented (Fig. 2) a straight line $y(t) = x(t)$ is the reference trajectory. During the first 15 seconds the flatness control law allows the drone to follow this line but as soon as $\theta_2 < 0.3$ the time-varying controller tries to stop it. It can be noticed that this last control law converges to $\begin{pmatrix} \dot{x} \\ \dot{y} \\ \psi_1 \end{pmatrix} = 0$.

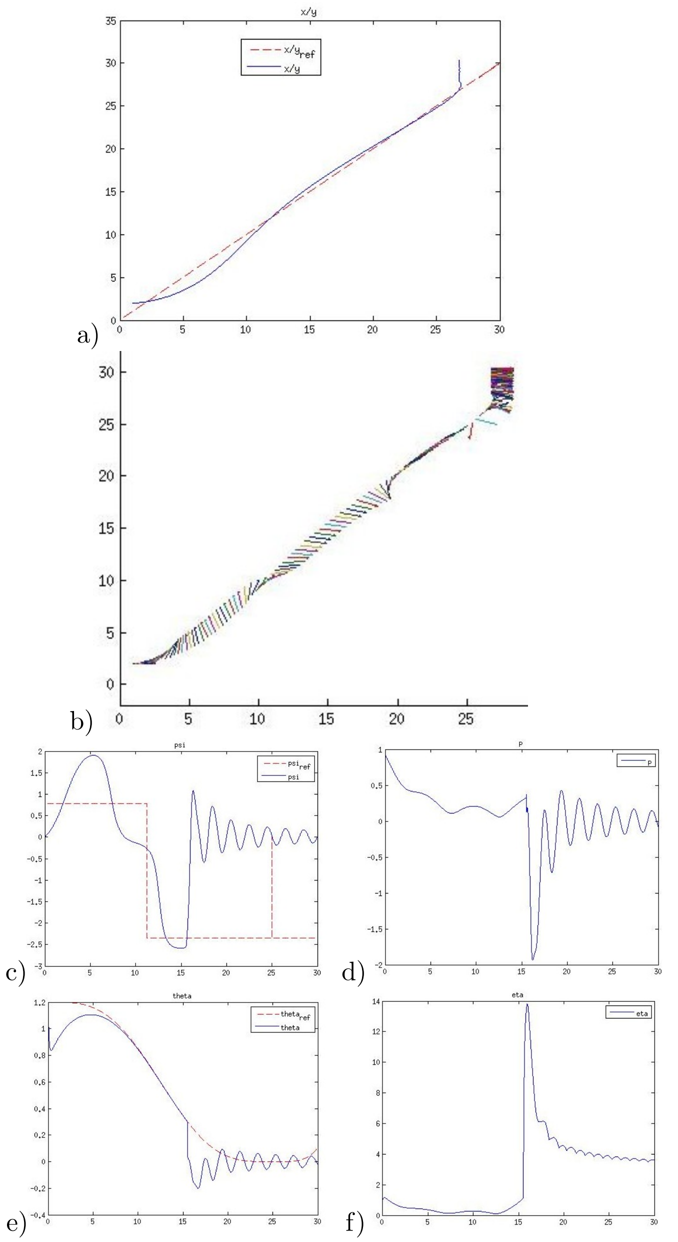

Fig. 2. Simulation results. (a) Trajectory in the plane X/Y in meter, (b) Trajectory in the plane X/Y with the yaw angle symbolized by the arrow in meter, (c) ψ_1 state in radian in function of time in seconds, (d) p state in Newton in function of time in seconds, (e) θ_2 state in radian in function of time in seconds, (f) η state in Newton in function of time in seconds.

5. CONCLUSIONS

Finally these two combined approaches by a flat and time-varying method can solve the tracking and point stabilization problem but with some restrictions: $\eta \neq 0$ and $\theta_2 \neq 0$ for tracking and the convergence for the point stabilization is slow. Our future work on this thematic will be to experiment them with a motion capture system to give us the state in real-time. Moreover we will explore other control theories like the transverse function approach for "practical" stabilization and homogeneous control laws.

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