Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

IDENTIFICATION OF HAMMERSTEIN-WIENER SYSTEMS WITH BACKLASH INPUT NONLINEARITY BORDERED BY STRAIGHT LINES

[1] A. Brouri*, [2] F. Giri, [3] F. Ikhouane, [2] F.Z. Chaoui, [2] O. Amdouri

[1] *Moulay Ismail University, ENSAM, L2MC, Meknes, Morocco*

[2] *Université de Caen Basse-Normandie, GREYC UMR CNRS, 14032 Caen, France*

Universitat Politecnica de Catalunya, Barcelona, Spain

* corresponding author: brouri_adil@yahoo.fr

Abstract: Standard Hammerstein-Wiener models consist of a linear subsystem sandwiched by two memoryless nonlinearities. Presently, the linear subsystem may be parametric or not, continuous- or discrete-time. The input nonlinearity is allowed to be a memory operator of backlash type bordered by straight lines. The output nonlinearity may be noninvertible and is only supposed to be well approximated, within any subinterval belonging to the working interval, with a polynomial of unknown order and parameters. An optimal strategy is presented to identify the system nonlinearities and an identification approach is developed that provides estimates of the linear subsystem. The method involves easily generated excitation signals. Finally, all the suggested estimators are shown consistent.

Keywords: Hammerstein-Wiener models, Backlash operator, Backlash-inverse operator, Frequency identification.

1. INTRODUCTION

Hammerstein-Wiener models consist of a linear dynamic block sandwiched by two nonlinear elements (Fig. 1). Clearly, this model structure is a generalization of Hammerstein and Wiener models and so it is expected to feature a superior modelling capability. This has been confirmed by several practical applications e.g. RF power amplifier modelling (Taringou et al., 2010), ionospheric dynamics (Palanthandalam-Madapusi et al., 2005), and chemical processes (Giri and Bai, 2010). As a matter of fact, Hammerstein-Wiener systems are more difficult to identify than the simpler Hammerstein and Wiener models. The complexity of the former lies in the fact that these systems involve two internal signals not accessible to measurements, whereas the latter only involve one. Then, it is not surprising that only a few methods are available that deal with Hammerstein-Wiener (HW) system identification. Identification methods for HW systems are generally classified into several families: iterative methods (e.g. Ni et al., 2012; Vörös, 2004), over-parameterized methods (see Bai, 2002; Wang et al., 2009). HW system identification is dealt given the assumption of invertibility of the Wiener part (e.g. Ni et al., 2013; Bai, 2002; Wang et al., 2009). More recently, a blind approach is aiming at estimating the model between the internal variables (Giri and Bai, 2010). Roughly, the iterative methods can lead to unsatisfactory results because they require a large amount of data and have local convergence properties which necessitates that a fairly accurate parameter estimates are available to initialize the search process. The over-parameterized methods involve a large number of parameters that are estimated. Despite numerous works reported for HW systems, the frequency identification problem is not yet fully studied.

Presently, the problem of identifying HW systems is tackled in the presence of a backlash operators bordered by straight lines (Fig. 2). The output nonlinearity is only supposed to be well approximated, within any subinterval belonging to the working interval, with a polynomial of unknown order and parameters. The order p and the parameters of the polynomial can vary from one subinterval to another. The memory nature of F[.] implies that the backlash output, at a given time t, is not uniquely determined by the input $u(t)$ at the same time. The major difficulty of the identification problem lies in the fact that the linear subsystem is of structure totally unknown and the internal signals of the system (i.e. x_i , x_o and $w(t)$) are not accessible to measurement. In view of these difficulties, it is not surprising that there are very few solutions available that deal with HW systems identification containing memory elements in the input nonlinearity (Giri et al., 2013). It is also worthy to note that the identification method involves easily generated excitation signals. The linear subsystem structure is presently totally unknown, unlike most previous works, where that subsystem is supposed to be a transfer function of known order (e.g. Bai, 2002; Ni et al., 2012; Wang et al., 2009; Schoukens et al., 2012). Also, these studies consider that the invertibility of output nonlinearity is a usual assumption. Presently, the identification problem was dealt with in two stages. The first stage is devoted to the estimation of a set of points belonging to the output nonlinearity and determining the parameters of the backlash borders. Once the input nonlinearity is identified, the linear subsystem frequency gain is determined using a frequency identification method involving backlash inversion (Yi Su and Stepanenko, 2000).

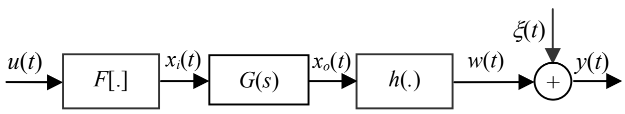

Fig. 1. Hammerstein-Wiener Model structure with a backlash input nonlinearity.

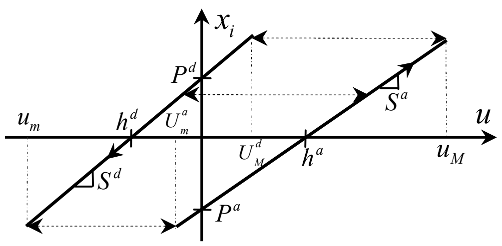

Fig. 2. Backlash bordered by straight lines.

To close this section, we give an outline of the paper. Section 2 formulates the problem and derives some preliminary results. The main results are given in Section 3 along with some remarks and proposition concerning the scheme applies to identify the system nonlinearities applied. The phase and gain identification of the linear block is coped with in Section4.

2. PROBLEM STATEMENT AND PRELIMINARIES

We are considering nonlinear systems that can be described by the Hammerstein-Wiener model (Fig. 1). The input nonlinearity is a Backlash operator bordered by two straight lines, having the characteristics (S^a, P^a) and (S^d, P^d) (Fig. 2), i.e. on the lateral borders, $x_i(t)$ holds one of two forms:

$$x_i(t) = S^a u(t) + P^a \quad \text{or} \quad x_i(t) = S^d u(t) + P^d \tag{1a}$$

Then, the above model is analytically described by:

$$x_i(t) = F[u](t) \tag{2a}$$

$$x_o(t) = g(t) * x_i(t) \tag{2b}$$

$$y(t) = h\left(x_o(t)\right) + \xi(t) \tag{2c}$$

where $g(t) = L^{-1}(G(s))$ and * refers to the convolution operation. The system input and output signals are accessible to measurement while the internal signals $x_i(t)$, $x_o(t)$ and $w(t)$ are not. The equation error $\xi(t)$ is a zero-mean stationary sequence of independent random variables and ergodic; it accounts for external noise. Accordingly, $G(s)$ is a transfer function, with impulse response $g(t)$, representing the system dynamics. Presently, $G(s)$ is allowed to be nonparametric and infinite order but is BIBO stable. Let:

$$U^d_M = (S^a / S^d)u_M + (P^a - P^d) / S^d \tag{3a}$$

$$U^a_m = (S^d / S^a)u_m + (P^d - P^a) / S^a \tag{3b}$$

denoting respectively the maximum and minimum inputs applied on the descendant and ascendant borders respectively (Fig. 2), for a given working interval $[u_m \;\; u_M]$. Obviously, if the signal $u(t)$ spans monotonically, in both senses, a sufficiently wide working interval then, the working point will span a closed backlash cycle, passing from one border to the other along two connecting horizontal paths (Fig. 3a). Accordingly, the working interval $[u_m \;\; u_M]$ is required to be sufficiently wide so that both borders can be actually attended by the backlash working point $(u(t), x_i(t))$ whenever $u(t)$ spans this interval monotonically in both senses. In case the working interval is not sufficiently large, the resulting steady-state internal signal $x_i(t)$ will be constant i.e. the backlash working point $(u(t), x_i(t))$ will move along a horizontal segment (Fig. 3b). Then, the system output $y(t)$ becomes constant (up to noise) after a transient period. This observation can be based upon in practice to discard non-suitable choices of $[u_m \;\; u_M]$. On the other hand, the output nonlinearity is assumed to be approximated, within any subinterval belonging to the working interval, with a polynomial of unknown order and parameters. The output nonlinearity is not globally invertible but satisfied $h^{-1}(0) = 0$. Except for the above assumptions $h(.)$ is arbitrary; in particular, the order p and the parameters of $h(.)$ can vary from one subinterval to another. The identification problem consists in determining accurate estimates of the nonlinear operator parameters (S^a,P^a,S^d,P^d), a set of points belonging to the output nonlinearity, and the linear frequency gain $G(j\omega_k)$ with ω_k (k=1…m).

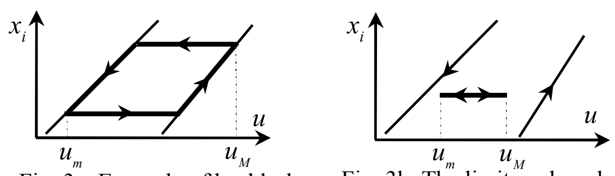

Fig. 3a. Example of backlash limit cycle.

Fig. 3b. The limit cycle reduces to a horizontal segment.

3. IDENTIFICATION OF SYSTEM NONLINEARITIES

In this section, an identification method is proposed to get estimates of the backlash borders and a set of points belonging to the output nonlinearity. The number of points N is arbitrary chosen. First, we note that the considered identification problem does not have a unique solution: if ($F[u]$, $G(s)$, $h(x_o)$) represents a solution then, any model of the form ($F[u]/k_1$,$G(s)/k_2$,$h(k_1k_2x_o)$) is also a solution (where k_1 and k_2 are any nonzero real). To solve this problem, it is suggested the following choice of the scaling factor:

$$k_1 = S^a \quad \text{and} \quad k_2 = G(0) \tag{4}$$

Then, the system to be identified is described by:

$$\bar{G}(s) = G(s) / G(0) \tag{5a}$$

$$\bar{F}[u] = F[u] / S^a \; ; \qquad \bar{h}(x) = h(S^a G(0)x) \tag{5b}$$

It is readily seen that: $\bar{G}(0) = 1$. Then, let:

$$\bar{S}^a = S^a / S^a = 1 \; ; \quad \bar{P}^a = P^a / S^a \tag{6a}$$

$$\bar{S}^d = S^d / S^a \; ; \quad \bar{P}^d = P^d / S^a \tag{6b}$$

designate respectively the parameters of ascendant and descendant borders of the modified Backlash, that will be identified. First, let $(u(t_0), x_i(t_0))$ be the initial backlash working point. To determine the estimations of $\bar{h}$ (.), we apply a set of constant inputs U_j (j=1…N), with:

$$u(t_0) = u_m < U_1 < \ldots < U_N = u_M \tag{7}$$

To make sure that all points $(U_j, F[U_j])$ belong to the ascendant straight, it is sufficient to ensure that $(U_1, F[U_1])$ belongs to this border. If $(U_1, F[U_1])$ moves along a horizontal segment (i.e. $F[U_1] = x_i(t_0)$), the system output $y(t)$ remains constant (up to noise). This observation can be based upon in practice to discard non-suitable choices of U_1.
Accordingly, apply the piecewise constant signal, for all $t \in [(j-1)MT_r \; jMT_r)$:

$$u(t) = U_j \quad \text{for} \quad j = 1 \ldots N \tag{8}$$

where the U_j satisfy (7), T_r should be comparable to the system rise time, and M> 1. Under these conditions, all points $(U_j, F[U_j])$ belong to the ascendant straight. Then, the internal signal $x_i(t)$ of the considered model (5a-b) is also piecewise constant signal, and is defined as follows, for all $t \in [(j-1)MT_r \; jMT_r)$:

$$\bar{X}_i^j = \bar{F}[U_j] = U_j + \bar{P}^a \quad \text{for} \quad j = 1 \ldots N \tag{9}$$

As the linear subsystem is asymptotically stable with unit static gain (i.e. $\bar{G}(0) = 1$), it follows that, on each interval $[(j-1)MT_r \; jMT_r)$ ($j = 1 \ldots N$), the steady-state of the internal signal $x_o(t)$ is constant i.e $x_o(t) \underset{t\to\infty}{\to} \bar{X}_o^j$, with:

$$\bar{X}_o^j \overset{def}{=} \bar{G}(0)\bar{X}_i^j = U_j + \bar{P}^a \quad \text{for} \quad j = 1 \ldots N \tag{10}$$

Subsequently, notice that the steady state of $w(t)$ is constant, noted W_j, and can simply be expressed as follows:

$$W_j \overset{def}{=} \bar{h}(U_j + \bar{P}^a) \quad \text{for} \quad j = 1 \ldots N \tag{11}$$

Finally, report that the steady state undisturbed output W_j can simply be estimated using the fact that $y(t) = w(t) + \xi(t)$ and $\xi(t)$ is zero-mean. Specifically, W_j can be recovered by averaging $y(t)$ on a sufficiently large interval. Let the nonlinearity $h^*(.)$ be defined as follows:

$$h^*(x) \overset{def}{=} \bar{h}(x + \bar{P}^a) \tag{12}$$

Hence, a number of points of the nonlinearity $h^*(.)$ can thus be accurately estimated.

Remark 1. a) The non-linearity $\bar{h}(.)$ is a more or less spread version (Giri et al., 2013), according the value of $S^a G(0)$, of $h(.)$ and satisfying $\bar{h}^{-1}(0) = 0$ (Fig. 4a).
b) The non-linearity $h^*(.)$ is a more or less spread version of $h(.)$ and horizontally shifted of $-\bar{P}^a$ (Fig. 4b). □

Consequently, knowing that $h(.)$ is assumed to be approximated, within any subinterval belonging to the working interval, with a polynomial and $h^{-1}(0) = 0$, the parameter $\bar{P}^a$ can be recovered (Fig. 4b). Then, the nonlinearity $\bar{h}(.)$ can be determined by using:

$$\bar{h}(x) = h^*(x - \bar{P}^a) \tag{13}$$

It now remains to determine the parameters of the descendant straight $\bar{S}^d$ and $\bar{P}^d$. To this end, one goes ahead realizing the descendant experiment series. Specifically, the system is successively excited with constant inputs with decreasing amplitudes $U_{N-1} < U_{N-2} < \ldots < U_1$. Following a similar argument as previously, it can be proved that the system asymptotic behavior is described by the following equations:

$$W_j = \bar{h}(\bar{S}^d U_j + \bar{P}^d); \quad j = N-1 \ldots 1 \tag{14}$$

The set of couples (U_j, W_j) all belonging to $\bar{h}(\bar{S}^d x + \bar{P}^d)$. Recall that the estimate of $\bar{h}(.)$ was carried out in the ascendant experimental stage, and getting benefit of the fact that the output nonlinearity is locally invertible, due to their polynomial nature. Then, $\bar{S}^d$ and $\bar{P}^d$ can be recovered by choosing two constant inputs U_p and $U_q \in \{U_{N-1}, \ldots, U_1\}$ in the invertibility zone of $\bar{h}(.)$.

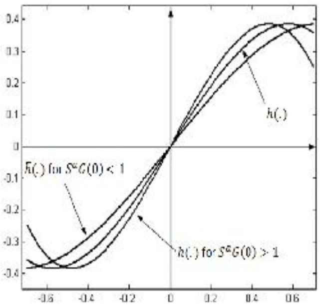

Fig. 4a. Example of function with spread versions.

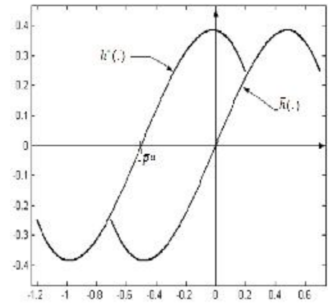

Fig. 4b. Comparison between the nonlinearities $h^*(.)$ and $\bar{h}(.)$.

These ideas are formalized in the estimator of Table 1.

Table 1. Nonlinearities Identification (NI)

1. (Initialization experiment) Apply the following signal:

$$u(t) = \begin{cases} u_M & for \quad t = 0 \\ u_m & for \quad t > 0 \end{cases} \tag{15}$$

2. (Data acquisition) Apply the piecewise constant signal analytically defined as follows for all $t \in \left[(j-1)MT_r \; jMT_r\right]$, $j = 1 \ldots N$:

$$u(t) = U_j; \text{ with } U_1 < \ldots < U_N = u_M \text{ and } U_1 \geq \bar{U} \tag{16}$$

Record the resulting output $y(t)$ for $t \in \left[0 \;\; NMT_r\right]$.

3. (Filtered output estimation) Compute the (undisturbed output) mean value on each interval $\left[(j-1)MT_r \; jMT_r\right]$:

$$\hat{W}_j(M) = \frac{1}{(M-1)T_r}\int_{(j-1)MT_r+T_r}^{jMT_r} y(t)dt \quad \text{for } j=1...N \tag{17}$$

Then, an exact estimation $\hat{h}^*_{M,N}(.)$ of $h^*(.)$ is obtained using the set of couples $\left(U_j, \hat{W}_j(M)\right)$ (estimates of N points all belonging to the curve of $h^*(.)$).

4. Determine the ascendant straight parameters of the backlash operator:

$$\hat{\bar{S}}^a = 1 \quad \text{and} \quad \hat{\bar{P}}^a_{M,N} = -\left(\hat{h}^*_{M,N}\right)^{-1}(0) \tag{18}$$

An exact estimation $\hat{\bar{h}}_{M,N}(.)$ of $\bar{h}(.)$ can be determined using:

$$\hat{\bar{h}}_{M,N}(x) = \hat{h}^*_{M,N}(x - \hat{\bar{P}}^a_{M,N}) \tag{19}$$

5. Apply the piecewise constant signal, analytically defined as follows, for all $t \in \left[jMT_r \; (j+1)MT_r\right]$, $j = N+1...2N-1$:

$$u(t) = U_j \quad \text{with} \quad U_{N+i} = U_{N-i} \quad \text{for} \quad 1 \le i \le N-1 \tag{20}$$

Applying (17), compute the corresponding filtered output $\hat{W}_j(M)$ for $j = N+1...2N-1$.

6. Select two constant inputs U_p and $U_q \in \{U_{N+1},...,U_{2N-1}\}$ in the invertibility zone of $\bar{h}(.)$. Calculate the estimates of $\bar{S}^d$ and $\bar{P}^d$:

$$\hat{\bar{S}}^d_{M,N} = \frac{\hat{\bar{h}}^{-1}_{M,N}\left(\hat{W}_q(M)\right) - \hat{\bar{h}}^{-1}_{M,N}\left(\hat{W}_p(M)\right)}{U_q - U_p} \tag{21a}$$

$$\hat{\bar{P}}^d_{M,N} = \frac{\hat{\bar{h}}^{-1}_{M,N}\left(\hat{W}_q(M)\right)U_p - \hat{\bar{h}}^{-1}_{M,N}\left(\hat{W}_p(M)\right)U_q}{U_p - U_q} \tag{21b}$$

Change the values of U_j ($j = N+1...2N-1$) in (20) if necessary.

Proposition 1. Consider the identification problem statement of Section 2. Let $\hat{\bar{h}}_N(.)$ and $\hat{h}^*_N(.)$ designate the estimates of $\bar{h}(.)$ and $h^*(.)$, respectively, as $M \to \infty$. Then, one has:

1) The estimate $\hat{h}^*_N(.)$ converge in probability to $h^*(.)$ (as $M \to \infty$) i.e.: $\lim_{N\to\infty} \hat{h}^*_N(.) = h^*(.)$ (*w.p.1*).

2) The estimate $\hat{\bar{h}}_N(.)$ converge in probability to $\bar{h}(.)$ (as $M \to \infty$) i.e.: $\lim_{N\to\infty} \hat{\bar{h}}_N(.) = \bar{h}(.)$ (*w.p.1*).

3) Let $\hat{\bar{F}}_{M,N}[.]$ denotes the estimate of $\bar{F}[.]$ using the NI estimator (Table 1). Then, $\hat{\bar{F}}_{M,N}[.]$ converge in probability to $\bar{F}[.]$ (as $(M,N) \to \infty$). □

Proof. It has already been noticed that, after the initialization experiment (step 1 of the NI procedure) and applying the input signal (16) for $t \in \left[0 \; NMT_r\right]$, with $U_1 < \ldots < U_N$ and make $\left|U_1 - u(t_0)\right|$ large, the backlash working point moves on the ascendant straight. One has, in the steady-state, from (9)-(11) and the asymptotic stability of linear subsystem that, for $t \in \left[0 \; NMT_r\right]$, the set of points $\left(U_j, W_j\right)$ ($j = 1...N$) occupying N positions on the trajectory of nonlinearity $h^*(.)$. On the other hand, using the fact that the noise $\xi(t)$ in (2c) is zero-mean:

$$\lim_{M\to\infty} \hat{W}_j(M) = \lim_{M\to\infty} \frac{1}{(M-1)T_r}\int_{(j-1)MT_r+T_r}^{jMT_r} y(t)dt$$

$$= \lim_{M\to\infty} \frac{1}{(M-1)T_r}\int_{(j-1)MT_r+T_r}^{jMT_r} w(t)dt \quad \text{for } j = 1\ldots N \tag{22}$$

Specifically, one has for $j = 1\ldots N$:

$$\lim_{M\to\infty} \hat{W}_j(M) = \bar{h}(U_j + \bar{P}^a) = h^*(U_j) \tag{23}$$

Then, we can conclude that:

$$\lim_{M\to\infty}\left(U_j, \hat{W}_j(M)\right) = \left(U_j, h^*(U_j)\right) \quad \text{for } j = 1\ldots N \tag{24}$$

Therefore, the set of points $\left(U_j, \hat{W}_j(M)\right)$ ($j = 1\ldots N$), obtained from the data collected on the time interval $\left[0 \; NMT_r\right]$, converge (in probability) to trajectory of $h^*(.)$ as $M \to \infty$. Furthermore, note that all input values U_j satisfy the condition $U_j \in [U^a_m \; u_M]$ ($j = 1\ldots N$). Finally, it is readily seen that $\hat{h}^*_N(.)$ converge in probability to $h^*(.)$, as $N \to \infty$, which proves the first part of Proposition.

One gets from (18) that: $\left(\hat{h}^*_{M,N}\right)^{-1}(0) = -\hat{\bar{P}}^a_{M,N}$. It follows from Part 1 that:

$$\lim_{N\to\infty}\left(\hat{h}^*_N\right)^{-1}(0) = \left(h^*\right)^{-1}(0) = -\bar{P}^a \tag{25}$$

Then, one has w.p.1:

$$\lim_{(M,N)\to\infty} \hat{\bar{P}}^a_{M,N} = -\left(h^*\right)^{-1}(0) = \bar{P}^a \tag{26}$$

Accordingly, it is readily obtained from Part 1, (19) and (26) that $\hat{\bar{h}}_N(.)$ converge in probability to $\bar{h}(.)$ (as $M \to \infty$). This proves the second part of proposition.

Finally, knowing that $\hat{\bar{S}}^a$ is exactly equal to unity, by using (21a-b) and (26), the direct results of second part complete the proof of the proposition. ■

4. LINEAR SUBSYSTEM IDENTIFICATION

In this section, a frequency identification method is proposed to obtain estimates of the complex gain at ω, whatever the frequencies $\omega > 0$.

a) Backlash-inverse operators.

Recall that a backlash-inverse operator is also a memory element characterized by a couple of functions (f_a, f_d), called its borders, and is denoted $BI(f_a, f_d)$. When submitted to an input signal $u(t)$, it generates an output signal $z(t)$ defined as follows:

$$z(t) = \begin{cases} f_a(u(t)) & \text{if } \dot{u}(t) > 0 \\ f_d(u(t)) & \text{if } \dot{u}(t) < 0 \end{cases} \tag{27}$$

This definition entails no condition on the couple (f_a, f_d) except for the obvious property:

$$|f_d(x)| < \infty \Leftrightarrow |f_a(x)| < \infty \text{, for all } x. \tag{28}$$

Fig. 5 shows an example of backlash-inverse operator bordered by straight lines, where the ascendant and descendant borders have the slopes $(S^a)^{-1}$ and $(S^d)^{-1}$ respectively.

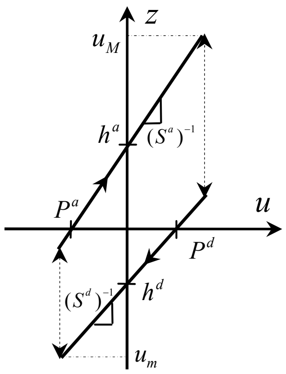

Fig. 5. Example of backlash-inverse operator bordered by straight lines.

b) Compound operators.

In the rest of this section, $X(f_a, f_d)$ and $Y(f_a, f_d)$ denotes any (backlash or backlash-inverse) operator and h any function. Then, one defines the composition $h \circ X(f_a, f_d)$ as follows:

$$(h \circ X(f_a, f_d))[u] = h(X(f_a, f_d)[u]) \tag{29}$$

where u is any signal whose values, $u(t)$ $(t \geq 0)$, belong to the domain of definition of the function pair (f_a, f_d). Similarly, one defines the composition $Y(h_a, h_d) \circ X(f_a, f_d)$ as follows:

$$(Y(h_a, h_d) \circ X(f_a, f_d))[u] = Y(h_a, f_d)(X(h_a, f_d)[u]) \tag{30}$$

It is readily seen that (30) describes the series system composed of the two operators (Fig. 6).

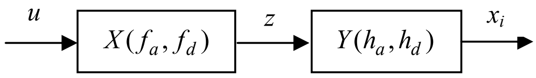

Fig. 6. Compound memory operators

Proposition 2 (Compositions involving memory operators). Let $B(f_a, f_d)$ denotes a backlash operator, with (f_a, f_d) its borders. Then, one has:

1) For any function $h : \mathbf{R} \to \mathbf{R}$:

$h \circ B(f_a, f_d) = B(h_a \circ f_a, h_d \circ f_d)$

2) Suppose f_a, f_d are similarly monotonic on some domain $D_f \subset \mathbf{R}$. then, one has on $f_a^{-1}(D_f) \cap f_d^{-1}(D_f)$:

$X(f_a, f_d) \circ BI(f_a^{-1}, f_d^{-1}) = I$, if f_a, f_d are increasing

$X(f_a, f_d) \circ BI(f_d^{-1}, f_a^{-1}) = I$, if f_a, f_d are decreasing

for $X \in \{B, BI\}$ with I being the identity operator. That is, for any signal $u(t)$ such that $u(t) \in f_a^{-1}(D_f) \cap f_d^{-1}(D_f)$, one has:

$(B(f_a, f_d) \circ BI(f_a^{-1}, f_d^{-1}))[u](t) = u(t)$, for all $t \geq 0$ □

Proof (Outline). Part 1 is easily checked using the definitions of Backlash operators, (27) and (29).

Owing to Part 2, it is well known that the property $B(f_a, f_d) \circ BI(f_a^{-1}, f_d^{-1}) = I$ holds in the case of similarly monotonic straight-line borders i.e. affine functions f_a, f_d. The proof can be found in many places, e.g. (Tao and Kokotovic, 1996), and does easily extend to the case of any similarly monotonic borders. Now, let us prove that one also has $BI(f_a, f_d) \circ BI(f_a^{-1}, f_d^{-1}) = I$. Consider any signal such that $u(t) \in f_a^{-1}(D_f) \cap f_d^{-1}(D_f)$ and let:

$$z(t) = BI(f_a^{-1}, f_d^{-1})[u](t), \; x_i(t) = BI(f_a, f_d)[z](t) \tag{31}$$

By definition (see (27)), one has:

$$z(t) = \begin{cases} f_a^{-1}(u(t)) & \text{if } \dot{u}(t) > 0 \\ f_d^{-1}(u(t)) & \text{if } \dot{u}(t) < 0 \end{cases} \tag{32a}$$

$$x_i(t) = \begin{cases} f_a(z(t)) & \text{if } \dot{z}(t) > 0 \\ f_d(z(t)) & \text{if } \dot{z}(t) < 0 \end{cases} \tag{32b}$$

As (f_a, f_d) are increasing on D_f, so are (f_a^{-1}, f_d^{-1}) on $f_a^{-1}(D_f) \cap f_d^{-1}(D_f)$. Now, suppose that $\dot{u}(t) > 0$. It immediately follows from (32a) that $z(t) = f_a^{-1}(u(t))$ and $\dot{z}(t) > 0$ which, together with (32b), implies that $x_i(t) = f_a(z(t)) = u(t)$.

A similar result can be established when $\dot{u}(t) < 0$. This proves that $BI(f_a, f_d) \circ BI(f_a^{-1}, f_d^{-1}) = I$ wherever (f_a, f_d) are monotonically increasing. The case of locally monotonic decreasing functions is dealt with similarly. □

c) Identification of the linear element

At this point, the input and output nonlinearities, $F[.]$ and $h(.)$, are known and the aim is to determine the transfer function $G(r)$ $(r = s \text{ or } z)$ of the linear subsystem.

In this respect, recall that $f_a(.)$ and $f_d(.)$ are straight lines, which implies that the Backlash borders are asymptotically monotonic. At this stage $f_a(.)$ and $f_d(.)$ (the parameters $\bar{S}^a$, $\bar{P}^a$, $\bar{S}^d$, and $\bar{P}^d$) are known and so one of such scalars can be explicitly determined. Then, Proposition 2 ensures that $F[.]$ is invertible and its right-inverse, denoted $F^{-1}[.]$ is equal either to $BI(f_a^{-1}, f_d^{-1})$ or $BI(f_d^{-1}, f_a^{-1})$, depending on the monotony sense of $f_a(.)$ and $f_d(.)$ (the sign of slopes $\bar{S}^a$ and $\bar{S}^d$, respectively). Then, one key idea is to neutralize the effect of $F[.]$ by placing its inverse (Fig. 5) as pre-compensator (Fig. 7) and excite the augmented system with input signals $u(t)$ that only take values in the domain where $F \circ F^{-1} = I$. Specifically, one has: $x_i(t) = u(t), \forall t$.

Then, the resulting system turns out to be a Wiener model. Roughly, the system identification can be dealt using several methods. One simple solution consists of getting benefit of the fact that the output nonlinearity $h(.)$ is locally invertible, due to their polynomial nature. Then, there exist an interval such that $h(.)$ is invertible. Let $[a \ b] \subset \mathbf{R}$ be any interval such that $h(.)$ is invertible on $[a \ b]$ and let its inverse on $[a \ b]$ be denoted $h^{-1}(.)$. Then, our second key idea is to choose the input signal $u(t)$ so that the observed output signal $w(t)$ is enforced to stay all the time in interval $[a \ b]$. This can be achieved by letting the working interval $[u_m \ u_M]$ be sufficiently large, whatever the nature of $u(t)$. In such an operation mode, the effect of the output nonlinearity can be cancelled by placing the element $h^{-1}(.)$ as post-compensator (Fig. 7). Doing so, the augmented system, including both the pre- and post-compensator, boils down to a linear system with transfer function $G(r)$. The fact that the input signal $u(t)$ is of arbitrary nature entails the possibility of choosing it to be persistently exciting making the problem of identifying $G(r)$ a trivial issue.

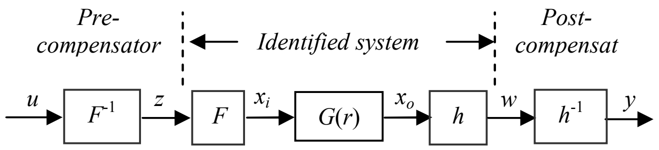

Fig. 7. The system to be identified augmented with pre- and post-compensator

5. SIMULATION

Due to space limitation, simulation results have been omitted. They will be presented at the conference.

6. CONCLUSIONS

We have developed a new two-stage identification method to deal with Hammerstein-Wiener systems in presence of backlash input nonlinearities and memoryless output nonlinearities. The originality of the present study lies in the fact that the linear subsystem is of structure totally unknown. On the other hand, the output nonlinearity may be noninvertible and is only supposed to be well approximated, within any subinterval belonging to the working interval, with a polynomial of unknown order and parameters.

Another feature of the method is the fact that the exciting signals are easily generated and the estimation algorithms can be simply implemented.

REFERENCES

Taringou, F., Hammi, O., Srinivasan, B., Malhame, R., and Ghannouchi, F.M. (2010). Behaviour modelling of wideband RF transmitters using Hammerstein-Wiener models. *IET Circuits Devices & Systems*, 4(4), 282-290.

Palanthandalam-Madapusi, H.J., Ridley, A.J., and Bernstein, D.S. (2005). Identification and prediction of ionospheric dynamics using a Hammerstein-Wiener model with radial basis functions. In *Proceedings of the American Control Conference*, 5052-5057.

Giri, F. and Bai, E.W. (2010). *Block-oriented Nonlinear System Identification*. Springer, 1 edition.

Giri, F., Brouri, A., Ikhouane, F., Chaoui, F.Z., Radouane, A. (2013). Identification of Hammerstein-Wiener Systems Including Backlash Input Nonlinearities. In *11th IFAC International Workshop on Adaptation and Learning in Control and Signal Processing*, 360-365. France.

Ni, B., Gilson, M., and Garnier, H. (2013). Refined instrumental variable method for Hammerstein-Wiener continuous-time model identification. *IET Control Theory and Applications*, 7 (9), 1276-1286.

Vörös, J. (2004). An iterative method for Hammerstein-Wiener systems parameter identification. *Journal of Electrical Engineering*, 55(11-12), 328-331.

Bai, E.W. (2002). A Blind Approach to the Hammerstein-Wiener Model Identification. *Automatica*, 38 (6), 967-979.

Wang, J., Chen, T., and Wang, L. (2009). A blind approach to identification of Hammerstein-Wiener systems corrupted by nonlinear-process noise. In *Proceedings of the 7th Asian Control Conference*, 1340-1345, Hong Kong, China.

Schoukens, M., Bai, E.W., and Rolain, Y. (2012). Identification of Hammerstein-Wiener Systems. In Proc. 16th IFAC Symp. Syst. Identification, 274-279. Brussels, Belgium.

Giri, F., Rochdi, Y., Radouane, A., Brouri, A., Chaoui, F.Z. (2013). Frequency Identification of nonparametric Wiener systems containing backlash nonlinearities. *Automatica*, 49, 124-137.

Tao G., P. Kokotovic. Adaptive Control of Systems with Actuator and Sensor Nonlinearities. John Wiley And Sons Ltd, US, 1996.

Yi Su, Y.T.C. and Stepanenko, Y. (2000). Adaptive Control of a class of nonlinear Systems preceded by an unknown backlash-like hysteresis. In *IEEE Conference on Decision and Control*, 1459-1464. Australia.