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Decoupled Nested LMI conditions for Takagi-Sugeno Observer Design

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Abstract: This work extends recent investigations on control design of continuous-time nonlinear models to non-quadratic observer design. The models under consideration are exactly rewritten in the Takagi-Sugeno form. By means of the Finsler's Lemma or a Tustin-like transformation, the progressively complex observer gains and the Lyapunov function are decoupled, thus providing the flexibility of using a quadratic Lyapunov functions while preserving the non-quadratic nature of the observer. Conditions obtained are expressed as linear matrix inequalities which are efficiently solved by convex optimization techniques. Examples are provided to show the effectiveness of the proposed approach.

1. INTRODUCTION

For many years, Takagi-Sugeno fuzzy models (TS) (Takagi and Sugeno 1985) have been an important topic of many research works in the control community due to their capacity to exactly represent an important class of nonlinear systems in a compact set of the state space. The TS representation is usually obtained via the sector nonlinearity approach (Taniguchi et al. 2001): it is an exact rewriting of the nonlinear model, not an approximation. A TS fuzzy model is composed of a set of linear models blended together with memberships functions (MFs) which contain the model nonlinearities and hold the convex sum property (Tanaka and Wang 2001). Taking advantage of their convex structure along with the direct Lyapunov method, TS models permit to obtain linear matrix inequality (LMI) conditions for stability analysis as well as for controller and observer design (Wang et al. 1996), (Tanaka et al. 1998). Getting LMI conditions is convenient since they are efficiently solved via convex optimization techniques (Boyd et al. 1994).

The aforementioned conditions are only sufficient due to several reasons: the way the MFs are dropped off or considered in the analysis, the model construction as well as the choice of the Lyapunov function (Feng et al. 2005), (Sala et al. 2005). Several works have been developed in order to tackle these sources of conservatism: diverse ways to obtain LMIs from nested convex sums (Tuan et al. 2001), (Liu and Zhang 2003), (Sala and Ariño 2007); different representations of the convex models such as descriptor (Guelton et al. 2009) or polynomial forms (Tanaka et al. 2009); more general Lyapunov functions such as piecewise (Johansson et al. 1999), line-integral (Rhee and Won 2006) and fuzzy (Tanaka et al. 2003), (Guerra and Vermeiren 2004).

In the continuous-time framework, non-quadratic Lyapunov functions have not met the development of the discrete-time domain (Guerra et al. 2009). The latter is due to the fact that the use of non-quadratic Lyapunov functions obliges to deal with the time-derivatives of the MFs (Blanco et al. 2001), a problem that has been considering in several works (Mozelli et al. 2009), (Bernal and Guerra 2010), (Lee et al. 2012).

This paper proposes a observer design scheme based on two former results: the Finsler's Lemma approach (Jaadari et al. 2012) and a Tustin-like transformation (Shaked 2001), (Márquez et al. 2013); it breaks the link between observer gains and the Lyapunov function.

This paper is organized as follows. Section 2 presents the TS model obtained by the sector nonlinearity methodology, provides basic notation and useful properties. In section 3 the main result in this paper is developed: it considers Finsler-based and Tustin-like approaches via a quadratic Lyapunov function for TS observers; moreover, thanks to a scheme of nested convex sums, it produces progressively more relaxed results. Section 4 gives some examples to illustrate the effectiveness of the proposed approaches, and finally, Section 5 briefs the paper results and discusses future work on the subject.

2. DEFINITIONS AND NOTATIONS

Consider a nonlinear model of the form

$$\begin{aligned} \dot{x}(t) &= f\left(z(t)\right)x(t) + g\left(z(t)\right)u(t) \\ y(t) &= e\left(z(t)\right)x(t) \end{aligned} \tag{1}$$

with $f(\cdot)$, $g(\cdot)$ and $e(\cdot)$ being nonlinear functions, $x(t) \in \mathbb{R}^n$ the state vector, $u(t) \in \mathbb{R}^m$ the input vector,

$y(t)\in\mathbb{R}^o$ the output of the system, and $z(x(t))\in\mathbb{R}^p$ the premise vector assumed to be bounded and smooth in a compact set C of the state space including the origin.

Let $z_j(\cdot)\in\left[\underline{z_j},\overline{z_j}\right]$, $j\in\{1,\cdots,p\}$ be the set of bounded nonlinearities in (1) belonging to C. Employing the sector nonlinearity approach (Taniguchi et al. 2001), the following weighting functions can be constructed

$$w_0^j(\cdot)=\frac{\overline{z_j}-z_j(\cdot)}{\overline{z_j}-\underline{z_j}},\quad w_1^j(\cdot)=1-w_0^j(\cdot),\ j\in\{1,\cdots,p\}. \tag{2}$$

From the previous weights, the following MFs are defined:

$$h_i=h_{1+i_1+i_2\times 2+\cdots+i_p\times 2^{p-1}}=\prod_{j=1}^{p}w_{i_j}^j(z_j) \tag{3}$$

with $i\in\{1,\cdots,2^p\}$, $i_j\in\{0,1\}$. These MFs satisfy the convex sum property $\sum_{i=1}^{r}h_i(\cdot)=1$, $h_i(\cdot)\geq 0$ in C. Where convenient, convex sums will be denoted as $\Upsilon_z=\sum_{i=1}^{r}h_i(z(t))\Upsilon_i$, their inverse as $\Upsilon_z^{-1}=\left(\sum_{i=1}^{r}h_i(z(t))\Upsilon_i\right)^{-1}$, and with extended indexes as $\Upsilon_{\overline{z}}=\sum_{i_1=1}^{r}\sum_{i_2=1}^{r}\cdots\sum_{i_q=1}^{r}h_{i_1}(z(t))h_{i_2}(z(t))\cdots h_{i_q}(z(t))\Upsilon_{i_1i_2\cdots i_q}$.

Based on the previous definitions, an exact representation of (1) in C is given by the following continuous-time T-S model:

$$\begin{aligned}\dot{x}(t)&=\sum_{i=1}^{r}h_i(z(t))\left(A_ix(t)+B_iu(t)\right)=A_zx(t)+B_zu(t)\\ y(t)&=\sum_{i=1}^{r}h_i(z(t))C_ix(t)\end{aligned} \tag{4}$$

with $r=2^p\in\mathbb{N}$ representing the number of linear models and (A_i,B_i,C_i), $i=1,\ldots,r$ a set of matrices of proper dimensions.

For brevity, an asterisk $(*)$ for inline expressions denotes the transpose of the terms on its left-hand side; for matrix expressions denotes the transpose of its symmetric block-entry. Should a matrix expression be involved with symbols "$<$" and "$>$", they will stand for negative and positive-definiteness, respectively. When convenient, arguments will be omitted.

The following properties will be used to develop the main results:

Property 1 (Schur complement): Let $P\in R^{n\times n}:P=P^T>0$, $X\in R^{m\times n}$ a full rank matrix, and $Q\in R^{n\times n}$, then (Boyd et al. 1994):

$$\begin{cases}Q-X^TP^{-1}X>0\\ P>0\end{cases}\Leftrightarrow\begin{bmatrix}Q & (*)\\ X & P\end{bmatrix}>0 \tag{5}$$

Property 2: Given $P=P^T>0$, then

$$Q^TP^{-1}Q\geq Q^T+Q-P \tag{6}$$

Property 3 (Finsler's Lemma): Let $x\in\mathbb{R}^n$, $Q=Q^T\in\mathbb{R}^{n\times n}$, and $R\in\mathbb{R}^{m\times n}$ such that $rank(R)<n$; the following expressions are equivalent:

a) $x^TQx<0$, $\forall x\in\left\{x\in\mathbb{R}^n:x\neq 0,Rx=0\right\}$

b) $\exists X\in\mathbb{R}^{n\times m}:Q+XR+R^TX^T<0$.

It is well-known that TS-LMI based controller design usually leads to inequalities containing multiple nested convex sums. For instance, given matrix expressions $\Upsilon_{i_0i_1\ldots i_q}$, $i_0,i_1,\ldots,i_q\in\{1,\ldots,r\}$, the following inequality may arise:

$$\sum_{i_0=1}^{r}\sum_{i_1=1}^{r}\sum_{i_2=1}^{r}\cdots\sum_{i_q=1}^{r}h_{i_0}(z(t))h_{i_1}(z(t))\ldots h_{i_q}(z(t))\Upsilon_{i_0i_1\ldots i_q}<0 \tag{7}$$

The sign of such expressions should be established via LMIs, which implies that the MFs therein should be adequately dropped-off: conditions thus obtained will be therefore only sufficient. This is why selecting a proper way to perform this task is important to reduce conservatism. When double sums are involved ($q=1$), a good compromise for guaranteeing (7) without adding slack variables is given by the following lemma:

Relaxation 1 (Tuan et al. 2001): Let $\Upsilon_{i_0i_1}$, $i_0,i_1\in\{1,\cdots,r\}$ be matrices of the same size. Condition (7) is verified for $q=1$ if :

$$\begin{aligned}&\Upsilon_{i_0i_0}<0,\quad\forall i_0\in\{1,\cdots,r\}\\ &\frac{2}{r-1}\Upsilon_{i_0i_0}+\Upsilon_{i_0i_1}+\Upsilon_{i_1i_0}<0,\quad\forall(i_0,i_1)\in\{1,\cdots,r\}^2,i_0\neq i_1\end{aligned}. \tag{8}$$

Should more than two nested convex sums be involved, a generalization of the sum relaxation in (Tanaka et al. 1998) will be used (Sala and Ariño 2007):

Relaxation 2 (Sala and Ariño 2007): Let $\Upsilon_{i_0i_1\ldots i_q}$, $i_0,i_1,\ldots,i_q\in\{1,\ldots,r\}$ be matrices of the same size and $\mathrm{P}(i_0,i_1,\cdots,i_q)$ be the set of all permutations of the indexes i_0, i_1,..., i_q. Condition (7) is verified if:

$$\sum_{i_0i_1\cdots i_q\in\mathrm{P}(i_0,i_1,\ldots,i_q)}\Upsilon_{i_0i_1\cdots i_q}<0,\quad\forall(i_0,i_1,\ldots,i_q)\in\{1,\ldots,r\}^{q+1}. \tag{9}$$

3. OBSERVER DESIGN

As in (Jaadari et al. 2012) and (Márquez et al. 2013), Finsler's Lemma and Tustin-like transformation will be used to relax the link between the Lyapunov function and the observer design. A quadratic Lyapunov function will be considered; it will outline the way to involve the approaches in the desired decoupling.

The proposed observer of the TS model in (4) has the following structure:

$$\begin{aligned}\dot{\hat{x}}(t) &= A_z\hat{x}(t) + B_z u(t) + H_{\bar{z}}^{-1}K_{\bar{z}}\left(y(t) - \hat{y}(t)\right) \\ \hat{y}(t) &= C_z\hat{x}(t)\end{aligned} \tag{10}$$

with $\hat{x}(t) \in \mathbb{R}^n$ as the observer state, $e(t) = x(t) - \hat{x}(t)$ as the estimation error, $H_{\bar{z}} \in \mathbb{R}^{n\times n}$ and $K_{\bar{z}} \in \mathbb{R}^{o\times n}$ matrix functions of the premise vector to be designed in the sequel. Therefore, the estimation error dynamics is described as:

$$\dot{e}(t) = \left(A_z - H_{\bar{z}}^{-1}K_{\bar{z}}C_z\right)e(t). \tag{11}$$

3.1 Finsler's lemma

Consider the quadratic Lyapunov function (QLF) candidate with $P = P^T > 0$

$$V(x(t)) = e(t)^T Pe(t). \tag{12}$$

Theorem 1 (QLF, generalized observer design via Finsler's lemma): The estimation error model (11) with $H_{\bar{z}}$ and $K_{\bar{z}}$ is asymptotically stable if $\exists\varepsilon > 0$, and matrices $P = P^T > 0$, $H_{i_1 i_2\cdots i_q}$, and $K_{i_1 i_2\cdots i_q}$, $i_1,\ldots,i_q \in \{1,\ldots,r\}$ of proper dimensions such that (9) holds with

$$\Upsilon_{i_0 i_1 i_2\cdots i_q} = \begin{bmatrix} H_{i_1 i_2\cdots i_q}A_{i_0} - K_{i_1 i_2\cdots i_q}C_{i_0} + (*) & (*) \\ \begin{pmatrix} P - H_{i_1 i_2\cdots i_q}^T \\ +\varepsilon\left(H_{i_1 i_2\cdots i_q}A_{i_0} - K_{i_1 i_2\cdots i_q}C_{i_0}\right)\end{pmatrix} & -\varepsilon\begin{pmatrix} H_{i_1 i_2\cdots i_q} \\ +H_{i_1 i_2\cdots i_q}^T\end{pmatrix}\end{bmatrix}. \tag{13}$$

Proof: Consider the quadratic Lyapunov function in (12); its time-derivative will thus be negative if:

$$\dot{V} = \begin{bmatrix} e \\ \dot{e}\end{bmatrix}^T \begin{bmatrix} 0 & P \\ P & 0\end{bmatrix}\begin{bmatrix} e \\ \dot{e}\end{bmatrix} < 0 \tag{14}$$

all together with the following restriction

$$\begin{bmatrix} A_z - H_{\bar{z}}^{-1}K_{\bar{z}}C_z & -I\end{bmatrix}\begin{bmatrix} e \\ \dot{e}\end{bmatrix} = 0, \tag{15}$$

arising from (11). Inequality (14) under equality constraint (15) holds is equivalent to through Finsler's Lemma:

$$\begin{bmatrix} 0 & P \\ P & 0\end{bmatrix} + \begin{bmatrix} U \\ W\end{bmatrix}\begin{bmatrix} A_z - H_{\bar{z}}^{-1}K_{\bar{z}}C_z & -I\end{bmatrix} + (*) < 0. \tag{16}$$

Let $U = H_{\bar{z}}$ and $W = \varepsilon H_{\bar{z}}$ with $\varepsilon > 0$, then (16) yields

$$\begin{bmatrix} H_{\bar{z}}A_z - K_{\bar{z}}C_z + (*) & (*) \\ P - H_{\bar{z}}^T + \varepsilon\left(H_{\bar{z}}A_z - K_{\bar{z}}C_z\right) & -\varepsilon\left(H_{\bar{z}} + H_{\bar{z}}^T\right)\end{bmatrix} < 0. \tag{17}$$

Conditions (9) with $\Upsilon_{i_0 i_1 i_2\cdots i_q}$ defined as in (13) guarantee the inequality above, thus producing the desired result. □

Remark 1: Parameter ε has been introduced in the aforementioned development to naturally include the quadratic case. Effectively, with $H_{\bar{z}} = P$ and $K_{\bar{z}} = K_z$, the Schur complement of (17) satisfies the following property:

$$PA_z - K_zC_z + (*) + \frac{1}{2}\varepsilon\left(PA_z - K_zC_z\right)^T P^{-1}(*) < 0, \tag{18}$$

which proves the referred inclusion when $\varepsilon > 0$ is enough small.

3.2 Tustin-like transformation

Theorem 2 (QLF, generalized observer design via Tustin-like transformation): The estimation error model (11) with $H_{\bar{z}}$ and $K_{\bar{z}}$ is asymptotically stable if $\exists\varepsilon > 0$, and matrices $P = P^T > 0$, $H_{i_1 i_2\cdots i_q}$, and $K_{i_1 i_2\cdots i_q}$, $i_1,\ldots,i_q \in \{1,\ldots,r\}$ of proper dimensions such that (9) holds with

$$\Upsilon_{i_0 i_1 i_2\cdots i_q} = \begin{bmatrix} -P & (*) \\ \begin{pmatrix} -H_{i_1 i_2\cdots i_q} \\ -\varepsilon\left(H_{i_1 i_2\cdots i_q}A_{i_0} - K_{i_1 i_2\cdots i_q}C_{i_0}\right)\end{pmatrix} & \begin{pmatrix} P - H_{i_1 i_2\cdots i_q} \\ -H_{i_1 i_2\cdots i_q}^T\end{pmatrix}\end{bmatrix}. \tag{19}$$

Proof: Consider the quadratic Lyapunov function in (12); derivative $\dot{V}(x(t)) < 0$ is satisfied if:

$$P\left(A_z - H_{\bar{z}}^{-1}K_{\bar{z}}C_z\right) + (*) < 0. \tag{20}$$

Considering a small enough $\varepsilon > 0$, it is clear that the following condition is equivalent to (20):

$$P\left(A_z - H_{\bar{z}}^{-1}K_{\bar{z}}C_z\right) + (*) + \varepsilon\left(A_z - H_{\bar{z}}^{-1}K_{\bar{z}}C_z\right)^T P(*) < 0 \tag{21}$$

from which the next rewriting can be done multiplying by ε and adding $P - P$:

$$\begin{aligned}&\varepsilon P\left(A_z - H_{\bar{z}}^{-1}K_{\bar{z}}C_z\right) + (*) \\ &\quad + \varepsilon^2\left(A_z - H_{\bar{z}}^{-1}K_{\bar{z}}C_z\right)^T P(*) + P - P < 0\end{aligned}$$

or rewritten:

$$\left(I + \varepsilon\left(A_z - H_{\bar{z}}^{-1}K_{\bar{z}}C_z\right)^T\right)P(*) - P < 0 \tag{22}$$

Thus by Schur complement (22) is equivalent to:

$$\begin{bmatrix} P & (*) \\ I + \varepsilon\left(A_z - H_{\bar{z}}^{-1}K_{\bar{z}}C_z\right) & P^{-1}\end{bmatrix} > 0, \tag{23}$$

which after pre-multiplication by $\begin{bmatrix} I & 0 \\ 0 & H_{\bar{z}}\end{bmatrix}$ and post-multiplication by $\begin{bmatrix} I & 0 \\ 0 & H_{\bar{z}}^T\end{bmatrix}$ gives:

$$\begin{bmatrix} P & (*) \\ H_{\bar{z}} + \varepsilon\left(H_{\bar{z}}A_z - K_{\bar{z}}C_z\right) & H_{\bar{z}}P^{-1}H_{\bar{z}}^T\end{bmatrix} > 0. \tag{24}$$

Using the property (6) with $Q = H_{\bar{z}}$, it is clear that $H_{\bar{z}}P^{-1}H_{\bar{z}}^T \geq H_{\bar{z}} + H_{\bar{z}}^T - P$, which allows guaranteeing (24) if the following holds:

$$\begin{bmatrix} P & (*) \\ H_{\bar{z}} + \varepsilon\left(H_{\bar{z}} A_z - K_{\bar{z}} C_z\right) & H_{\bar{z}} + H_{\bar{z}}^T - P \end{bmatrix} > 0 . \quad (25)$$

But (25) holds if relaxation (9) is applied with $\Upsilon_{i_0 i_1 i_2 \cdots i_q}$ defined as in (19), which concludes the proof. □

Remark 2: The inclusion of the quadratic case is also guaranteed. Proof is direct with a Schur complement using $H_{\bar{z}} = P$, $K_{\bar{z}} = K_z$ and the fact that ε can be a small as possible.

Remark 3: Results in this work are parameter-dependent LMI; their result depend on the choice of ε. Nevertheless, it has been proved in (de Oliveira and Skelton 2001) and (Oliveira et al. 2011) that a logarithmically spaced family of values, for instance $\varepsilon \in \left\{10^{-6}, 10^{-5}, \ldots, 10^{6}\right\}$, is adequate to avoid an exhaustive search of feasible solutions, thus outperforming existing results.

4. EXAMPLES

In this section some examples are presented to show the effectiveness of the proposed observer design.

4.1 Example 1

Consider the following TS model:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i\left(z(t)\right)\left(A_i x(t) + B_i u(t)\right) \quad (26)$$

with $A_1 = \begin{bmatrix} 1 & 0 \\ 5+5a & 10-10b \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 5-5a & 10+10b \end{bmatrix}$, $C_1 = \begin{bmatrix} 1+a \\ 1+b \end{bmatrix}^T$, $C_2 = \begin{bmatrix} 1-a \\ 1-b \end{bmatrix}^T$, $i = 1,2$, $z_1 = \sin x_1$, $w_0^1 = \frac{1-\sin x_1}{2}$, $w_1^1 = 1 - w_0^1$, $h_1 = w_0^1$, $h_2 = w_1^1$ with $a \in [-1,1]$, and $b \in [-1,1]$.

The following results were obtained considering $\varepsilon \in \mathrm{E} = \left\{10^{-6}, \ldots, 10^{6}\right\}$.

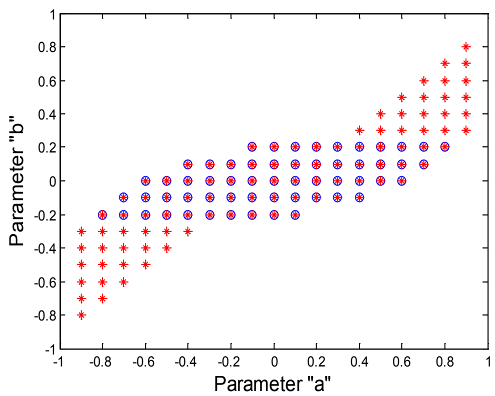

Fig. 1. Comparison: "$*$" for (13) with $q = 3$, "o" for (13) with $q = 1$.

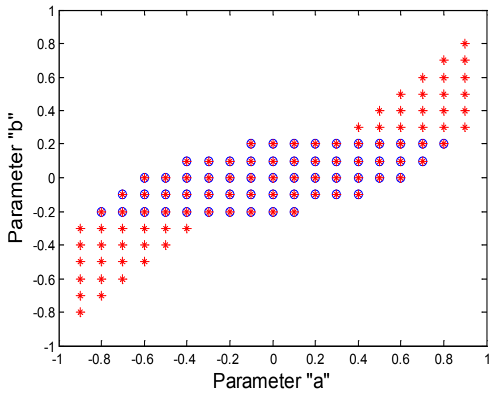

Fig. 2. Comparison: "$*$" for (19) with $q = 3$, "o" for (19) with $q = 1$.

Theorems 1 was compared considering two different values for q ($q = 1$ and $q = 3$). Fig. 1 shows that feasibility points of (13) with $q = 1$ are included in the solutions presented considering $q = 3$. Fig. 2 illustrate that solutions of (19) in theorem 2 considering $q = 3$ overcome solutions with $q = 1$.

Comparing results between Fig. 1 and Fig. 2, it is possible to observe that both approaches present the same feasibility region. Nevertheless, it is not possible to show theoretically that both approaches are equivalent or one included the other. They remain two different approaches to solve the problem.

4.2 Example 2

Consider the following TS model:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i\left(z(t)\right)\left(A_i x(t) + B_i u(t)\right) \quad (27)$$

with $A_1 = \begin{bmatrix} -1 & 1.5+a \\ 1.5 & -0.5-b \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & 1.5-a \\ 1.5 & -0.5+b \end{bmatrix}$, $C_1 = \begin{bmatrix} 1-b \\ 0 \end{bmatrix}^T$, $C_2 = \begin{bmatrix} 1+b \\ 0 \end{bmatrix}^T$, $i = 1,2$, $z_1 = x_1^2$, $w_0^1 = 1 - x_1^2$, $w_1^1 = 1 - w_0^1$, $h_1 = w_0^1$, with $a \in [-3,3]$, and $b \in [-3,3]$.

A non-PDC control law $u(t) = F_z G_z^{-1} x(t) + 0.1\sin(t)$ will be employed in order to stabilize the nonlinear system; the gains are calculated as in (Jaadari et al. 2012).

Theorem 2 with $q = 3$ was compared with conditions in (Bergsten et al. 2002) considering parameter $\varepsilon \in \mathrm{E} = \left\{10^{-6}, \ldots, 10^{6}\right\}$. Fig. 3 shows that solutions of conditions in (Bergsten et al. 2002) are included in the solutions of (19) with $q = 3$.

Using the Tustin-like approach in theorem 2 for state estimation (measured premises) and selecting the following parameters $q = 3$, $a = 1$, and $b = -2$, with $\varepsilon = 0.1$, a feasible solution has been found, it is not possible with conditions in (Bergsten et al. 2002).

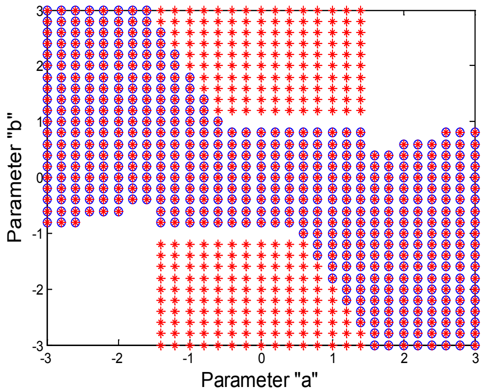

Fig. 3. Comparison: " $*$ " for (19) with $q=3$, "o" for conditions in *(Bergsten et al. 2002)*.

The gains for the observer and Lyapunov matrix are given by

$$P=\begin{bmatrix}3.263 & -1.446\\ -1.446 & 1.691\end{bmatrix}, K_{111}=\begin{bmatrix}12.680\\ -0.444\end{bmatrix}, \quad K_{112}=\begin{bmatrix}23.2469\\ 1.3349\end{bmatrix},$$

$$K_{121}=\begin{bmatrix}5.635\\ -0.490\end{bmatrix}, \quad K_{122}=\begin{bmatrix}-59.069\\ -3.511\end{bmatrix}, \quad K_{211}=\begin{bmatrix}5.635\\ -0.490\end{bmatrix},$$

$$K_{212}=\begin{bmatrix}74.642\\ -4.791\end{bmatrix}, \quad K_{221}=\begin{bmatrix}-74.642\\ -4.791\end{bmatrix}, \quad K_{222}=\begin{bmatrix}-41.483\\ 0.383\end{bmatrix},$$

$$H_{111}=\begin{bmatrix}4.132 & -0.718\\ -1.718 & 1.547\end{bmatrix}, \quad H_{112}=\begin{bmatrix}14.767 & 4.902\\ -26.003 & 121.072\end{bmatrix},$$

$$H_{121}=\begin{bmatrix}-7.699 & -2.406\\ 11.170 & -33.948\end{bmatrix}, \quad H_{122}=\begin{bmatrix}278.828 & -42.130\\ -6.069 & -334.907\end{bmatrix},$$

$$H_{211}=\begin{bmatrix}-2.003 & -2.406\\ 11.170 & -83.110\end{bmatrix}, \quad H_{212}=\begin{bmatrix}-112.002 & 20.350\\ 1.741 & 143.536\end{bmatrix},$$

$$H_{221}=\begin{bmatrix}-161.947 & 20.350\\ 1.741 & 195.714\end{bmatrix}, \quad H_{222}=\begin{bmatrix}4.344 & 0.067\\ -1.792 & 2.044\end{bmatrix}.$$

The estimation error for a trajectory of the states is presented in Fig 4.

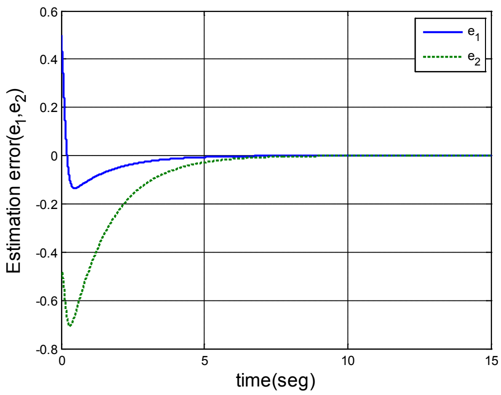

Fig. 4. Time evolution of the estimation error in Example 2.

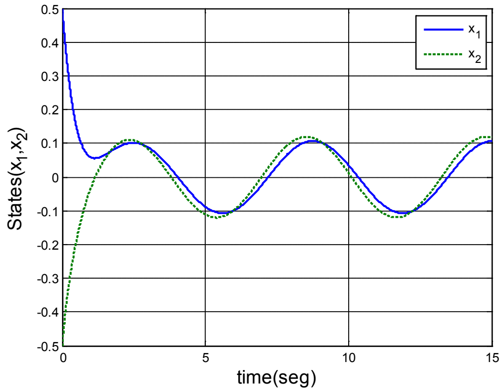

Fig. 5. Time evolution of the states in Example 2.

The initial condition are $[0.5 \quad -0.5]^T$, while the estimated ones are $[0 \quad 0]^T$. The time evolution of the states is shown in Fig. 5. It is clear to observe that the estimation error goes to zero despite the fact that the states remain oscillating.

5. CONCLUSIONS

Novel approaches for observer design for continuous-time nonlinear models have been reported. Taking advantage of a convex rewriting of the model (TS form) as well as the Finsler's Lemma or a Tustin-like transformation, the observer design has been decoupled from the quadratic Lyapunov function it is based on. It has been shown that the proposed decoupling introduces progressively better results thanks to a nested convex structure. Some examples have been given to illustrate the usefulness of the new schemes.

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