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Hidden periodic oscillations in drilling system driven by induction motor

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Abstract: This work is devoted to the investigation of oscillations in a drilling system using an induction motor with wound rotor as a drive. It is motivated by the problems of drilling rig failures in the oil and gas industry. The study is based on a modified version of the mathematical model of a drilling rig proposed by scientists from the Eindhoven University of Technology. The model of drilling rig developed in this work takes into account full description of the rotor geometry of induction motor. It is shown that such complex effects as hidden oscillations may appear in this kind of systems. To damper these oscillations a control strategy based on changing the external additional resistance in the rotor circuit is suggested.

Keywords: Drilling system, drill-string failures, induction motor, wound rotor, hidden oscillations, hidden attractor, multistability, coexistence of attractors

1. INTRODUCTION

Drilling systems are widely used for oil and gas exploration and production. The breakdowns of drilling rigs quite often occur while drilling. They lead to considerable time and costs. In order to reduce the number of breakdowns it is important to study oscillations appearing in drilling systems during the drilling process.

A schematic view of a typical drilling rig is shown in Fig. 1. The main constructive elements of drilling rig are hoisting system, motor, rotary table, and drill-string. The drill-string consists of three parts, namely, drill pipe, drill collar, and drill bit. The drilling system creates a borehole by the bit containing a cutting tool at the free end. The torque driving the drill bit is produced at the surface by a motor which is connected to the rotary table via transmission. [1]

During the drilling operations the drill-string undergoes different types of vibrations, which are classified depending on the direction they occur: torsional (rotational), axial (longitudinal), and lateral (bending) vibrations. Many researches are devoted to vibrations in drill-string systems (see, e.g., Brett (1992); Germay (2002); Jansen and Van den Steen (1995); Kreuzer and Kust (1996a,b); Keuzer and Kust (1997); Kust (1998); Kyllingstad and Halsey (1988); Jansen (1991); Leine (2000); Leine and van Campen (2002); Mihajlovic et al. (2005, 2004a,b); Van den Steen (1997)). In this work only torsional vibrations of the drill-string are considered, since they are regarded as the most damaging type of vibrations appearing in drilling system (Omojuwa et al. (2011); Rajnauth (2003)). They may cause failures of the drill-string or even the drilling system itself that lead to unacceptably high cost and time losses for the drilling industry.

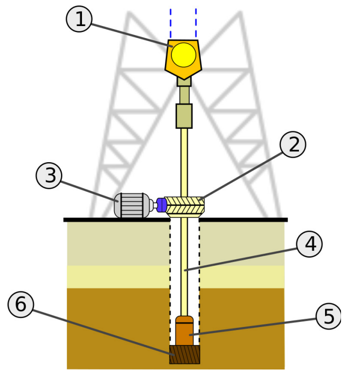

Fig. 1. Schematic view of drilling rig: 1 – hoisting system, 2 – rotary table, 3 – motor, 4 – drill pipe, 5 – drill collar, 6 – drill bit

To study the torsional dynamics, various mathematical models of drilling systems have been proposed and studied by many researchers (see, e.g., Germay et al. (2009); Palmov et al. (1995); Khulief et al. (2007); de Bruin et al. (2009); Mihajlovic et al. (2004a, 2006); Mihajlovic (2005)). The most analytically and numerically studied model and its modifications were developed in the works of de Bruin

[1] There are three types of rotating drilling systems (see Short (1983); Baker (2001)) The bit can be rotated with a rotary table, a top drive or by a slim downhole motor. The type of drilling rig with rotary table considered in this article is widely used nowadays since it makes it possible to have a greater torque on a drill bit than in other types of rigs. Nowadays the length of the drill-pipe in drilling rigs used in the oil and gas industry is usually varied from 1 to 8 km, while the diameter of the drill-pipe is several tens of centimeters (Baker (2001); Mihajlovic (2005)).

et al. (2009); Mihajlovic et al. (2004a, 2006); Mihajlovic (2005). A simple mathematical model of drilling system driven by DC motor was constructed and numerical analysis of this model was carried out in the works of Mihajlovic et al. (2004a); de Bruin et al. (2009). However DC motors require power sources of constant current and contain compound constructive elements (for example, collector), for which additional maintenance is necessary. The induction motors do not have these disadvantages, therefore, they are often used as the drivers in drilling rigs (see, for example, Hild (1934); Staege (1936); Hall and Shumway (2009)). It allows one to improve the reliability of the system.

In this article a drilling rig driven by an induction motor with wound rotor is studied. It is a modified version of the model suggested by researchers from the Eindhoven University of Technology (see de Bruin et al. (2009); Mihajlovic et al. (2006); Mihajlovic (2005)). The use of the induction motor with wound rotor as a drive in the drilling rig allows one to introduce the rheostat (variable external resistance) in the rotor circuit. In this case a control strategy by means of changing the external resistance is suggested in order to avoid hidden oscillations [2].

2. INDUCTION MOTOR WITH A WOUND ROTOR

Let us develop a mathematical model of induction motor for describing the drive part of drilling system. Unlike well-known mathematical models of induction machines (see, for example, (White and Woodson, 1968; Leonhard, 2001; Khalil and Grizzle, 2002; Marino et al., 2010)), the obtained below model completely takes into account rotor geometry (rotor winding configuration).

Induction machines have two main parts: stationary stator and rotating rotor. The windings are placed in the stator and rotor slots. The stator winding connected to the alternate current supply produces a rotating magnetic field.

Consider an induction motor with a wound rotor shown in Fig. 2. In the simplest case a wound rotor winding consists of three coils. Each coil contains several turns of insulated wire. Some ends of coils a, b, c (Fig. 3) are connected to the rotor itself at one point o. Another free ends of coils a', b', c' are connected to slip rings, mounted on the rotor shaft and isolated from it and each other. The brushes are resting on slip rings. The brushes, sliding over the surfaces of rotor rings, always have an electric contact with them and are connected, thus, with the rotor windings. The rotor winding can be either short-circuited or connected with other external devices through the brushes. Such devices are often used for a speed control of induction motors with wound rotor. Furthermore, the rotor winding is connected to a rheostat, which acts as a variable resistance in this case (Fig. 2).

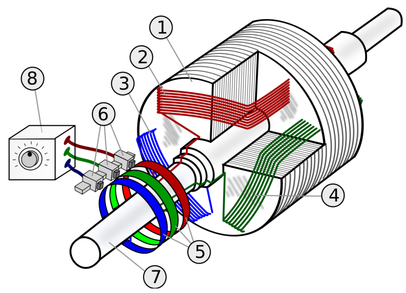

Fig. 2. Wound rotor with rheostat: 1 – rotor core, 2 – first coil with current i_1, 3 – second coil with current i_2, 4 – third coil with current i_3, 5 – slip rings, 6 – brashes, 7 – shaft, 8 – rheostat

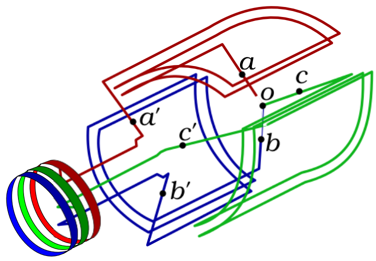

Fig. 3. Winding of wound rotor with slip rings

The classical derivation of the expressions for currents in the rotor winding and the electromagnetic torque of induction motor are based on the following simplifying assumptions (see Popescu (2000); Leonhard (2001); Skubov and Khodzhaev (2008)):

(1) It is assumed that the magnetic permeability of stator and rotor steel [3] is equal to infinity. This assumption makes it possible to use the principle of superposition for the determination of magnetic field, generated by stator;
(2) one may neglect energy losses in electrical steel, i.e., motor heat losses, magnetic hysteresis losses, and eddy-current losses;
(3) the saturation of rotor steel is not taken into account, i.e. the current of any force can run in rotor winding;
(4) one may neglect the effects, arising at the ends of rotor winding and in rotor slots, i.e., one may assume that a magnetic field is distributed uniformly along a circumference of rotor.

[2] An oscillation in a dynamical system can be easily localized numerically if initial conditions from its open neighborhood lead to long-time behavior that approaches the oscillation. Thus, from a computational point of view it is natural to suggest the following classification of attractors, based on the simplicity of finding basin of attraction in the phase space: *an attractor is called a hidden attractor if its basin of attraction does not intersect with small neighborhoods of equilibria, otherwise it is called a self-excited attractor* (Leonov and Kuznetsov, 2011; Leonov et al., 2011; Leonov and Kuznetsov, 2013). Self-excited attractors can be localized numerically by *standard computational procedure*, in which after a transient process a trajectory, started from a point of unstable manifold in a neighborhood of equilibrium, traces the state of oscillation and therefore it can be easily identified. In contrast, for numerical localization of hidden attractors it is necessary to develop special analytical-numerical procedures that allow one to determine the initial data from the basin of attraction for numerical procedure. For example, hidden attractors are attractors in the systems with no-equilibria or with the only stable equilibrium (a special case of multistability and coexistence of attractors); they arise in the study of well-known fundamental problems such as 16th Hilbert problem, Aizerman & Kalman conjectures and in applied research of Chua circuits, phase-locked loop based circuits, aircraft control systems and others (Kuznetsov et al., 2010; Bragin et al., 2011; Leonov et al., 2012; Kuznetsov et al., 2013; Andrievsky et al., 2013; Leonov and Kuznetsov, 2013).

[3] Usually both stator and rotor are made of laminated electrical steel.

Let us make an additional assumption: [4]

(5) stator windings are fed from a powerful source of sinusoidal voltage.

Then, following the works of Adkins (1957); White and Woodson (1968); Skubov and Khodzhaev (2008), by the latter assumption, the effect of rotor currents on stator currents may be ignored. Thus, the stator produces a uniformly rotating magnetic field with a constant in magnitude induction. So, it can be assumed that the magnetic induction vector B is constant in magnitude and rotates with a constant angular velocity n_1. This assumption goes back to the classical ideas of N. Tesla and G. Ferraris and allows one to consider the dynamics of induction motor from the point of view of its rotor dynamics (Leonov (2006)).

Introduce the uniformly rotating coordinates, rigidly connected with the magnetic induction vector B, and consider the motion of wound rotor in this coordinate system. Using the approach suggested in the works Leonov (2006); Leonov et al. (2013, 2014), we obtain the system of differential equations of wound rotor induction motor with the rheostat in the rotor circuit:

$$\begin{aligned} &J\ddot{\theta} = nBS\sum_{k=1}^{3} i_k \sin\big(\theta + \frac{2(k-1)\pi}{3}\big) - M_l,\\ &L\dot{i_1} + (R+r)i_1 = -nBS\dot{\theta}\sin\theta,\\ &L\dot{i_2} + (R+r)i_2 = -nBS\dot{\theta}\sin\big(\theta + \frac{2\pi}{3}\big),\\ &L\dot{i_3} + (R+r)i_3 = -nBS\dot{\theta}\sin\big(\theta + \frac{4\pi}{3}\big). \end{aligned} \tag{1}$$

where n – the number of turns in each coil; B – an induction of magnetic field; S – the area of one turn of the coil, θ – mechanical angle of rotation of rotor; i_k – currents in coils; R – the resistance of each coil; r – variable external resistance; L – the inductance of each coil; J – the moment of inertia of the rotor; M_l – load torque. Detailed derivation of (1) can be found in Leonov et al. (2014).

In the work of Leonov et al. (2013) the analysis of equations (1) is performed. The case of constant load torque is considered. A region of initial data, under which the induction motor with wound rotor pulls in synchronism, is obtained by Lyapunov functions and the modified nonlocal reduction method. Numerical analysis showed that in the case of constant load torque outside this region the system has no oscillating solutions. Further it will be shown that the hidden oscillations appearing in drilling rig used an induction motor with wound rotor as a drive can be eliminated by regulation of the external resistance.

3. DRILLING SYSTEM

In the works of Mihajlovic et al. (2004a); de Bruin et al. (2009) a double-mass mathematical model of drilling system is studied by researchers from the Eindhoven University of Technology. The mathematical model is based on an experimental setup. It consists of upper and lower discs connected with each other by a steel string. The upper disc is actuated by a DC motor and there is also a brake device which is used for modeling of the friction force acting on the lower disc (see Fig. 4). This model is described by the following differential equations

$$\begin{aligned} &J_u\ddot{\theta}_u + k_\theta(\theta_u - \theta_l) + b(\dot{\theta}_u - \dot{\theta}_l) + T_{fu}(\dot{\theta}_u) - k_m v = 0,\\ &J_l\ddot{\theta}_l - k_\theta(\theta_u - \theta_l) - b(\dot{\theta}_u - \dot{\theta}_l) + T_{fl}(\dot{\theta}_l) = 0. \end{aligned} \tag{2}$$

Here θ_u and θ_l – angular displacements of upper and lower discs, J_u and J_l – constant inertia torques, b – rotational friction (damping), k_θ – torsional spring stiffness, k_m – the motor constant, v – a constant input voltage, $T_{fu}(\dot{\theta}_u)$ and $T_{fl}(\dot{\theta}_l)$ – friction torques acting on the upper and the lower discs. The torque $T_{fl}(\dot{\theta}_l)$ appears due to the friction between the drill bit (lower disc) and the bedrock to be drilled. $T_{fu}(\dot{\theta}_u) - k_m v$ is a result of influence of driving part on the drill-string.

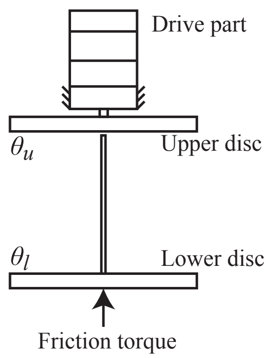

Fig. 4. Drilling system actuated by induction motor

The driving part of the model, considered above, is a linear combination of constant input voltage and friction torque $T_{fu}(\dot{\theta}_u)$ acting on the upper disc, which is built according to computational experiments. Following the patents of Hild (1934); Staege (1936); Hall and Shumway (2009), let us extend this model, introducing the equations of induction motor with wound rotor described above (Leonov et al. (2014); Kiseleva (2013)). Note that later on it is convenient to assume that θ_u and θ_l are angular displacements of the upper and lower discs relative to the magnetic field, which rotates with the speed ω. This allows one to obtain a system of equations (using the equations (2) and (1)), which takes into account more detailed dynamics of motor, namely

$$\begin{aligned} &J_u\ddot{\theta}_u + k_\theta\big(\theta_u - \theta_l\big) + b\big(\dot{\theta}_u - \dot{\theta}_l\big) -\\ &\qquad - nBS\sum_{k=1}^{3} i_k \sin\big(\theta_u + \frac{2(k-1)\pi}{3}\big) = 0,\\ &J_l\ddot{\theta}_l - k_\theta(\theta_u - \theta_l) - b(\dot{\theta}_u - \dot{\theta}_l) + T_{fl}(\omega + \dot{\theta}_l) = 0,\\ &L\dot{i}_1 + (R+r)i_1 = -nBS\dot{\theta}_u\sin\theta_u,\\ &L\dot{i}_2 + (R+r)i_2 = -nBS\dot{\theta}_u\sin(\theta_u + \frac{2\pi}{3}),\\ &L\dot{i}_3 + (R+r)i_3 = -nBS\dot{\theta}_u\sin(\theta_u + \frac{4\pi}{3}). \end{aligned} \tag{3}$$

Let us introduce the friction model suggested by the researchers from Eindhoven University of Technology (see Fig. 5, Mihajlovic et al. (2004a); de Bruin et al. (2009)):

[4] Without this assumption it is necessary to consider a stator, what leads to a more complicated derivation of equations and more complicated equations themselves, which are difficult for analytical and numerical analyzing.

$$T_{fl}(\omega+\dot\theta_l) \in \begin{cases} T_{cl}(\omega+\dot\theta_l)\mathrm{sign}(\omega+\dot\theta_l), & \omega+\dot\theta_l \neq 0 \\ [-T_0, T_0], & \omega+\dot\theta_l = 0, \end{cases} \quad (4)$$

where

$$T_{cl}(\omega+\dot\theta_l) = \frac{T_0}{T_{sl}}(T_{pl} + (T_{sl}-T_{pl})e^{-|\frac{\omega+\dot\theta_l}{\omega_{sl}}|^{\delta_{sl}}} + b_l|\omega+\dot\theta_l|). \quad (5)$$

Here T_0, T_{sl}, T_{pl}, ω_{sl}, δ_{sl}, and b_l – nonnegative coefficients. T_0 is an additional parameter for changing drilling medium. Note that by replacing in (3)

$$-nBS\sum_{k=1}^{3} i_k \sin\left(\theta_u + \frac{2(k-1)\pi}{3}\right)$$

with

$$k_m v - T_{fu}(\omega+\dot\theta_u) \text{ and } T_0 = T_{sl}$$

we will obtain the drilling system described in Mihajlovic et al. (2004a); de Bruin et al. (2009).

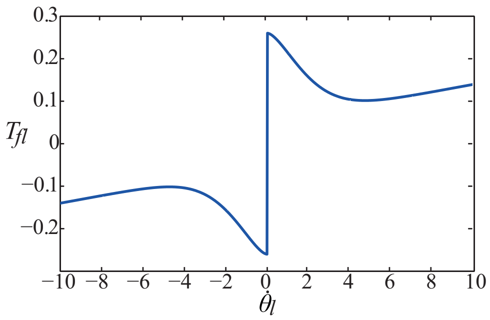

Fig. 5. Friction torque T_{fl}

Transform system (3) to more convenient form. Introduce the following nonsingular transformation of coordinates:

$$\omega_u = -\dot\theta_u,\ \omega_l = -\dot\theta_l,\ \theta = \theta_u - \theta_l,$$

$$x = -\frac{2}{3}\frac{L}{nSB}\sum_{k=1}^{3} i_k \sin\left(\theta_u + \frac{2(k-1)\pi}{3}\right),$$

$$y = -\frac{2}{3}\frac{L}{nSB}\sum_{k=1}^{3} i_k \sin\left(\theta_u + \frac{2(k-1)\pi}{3}\right),$$

$$z = i_1 - i_2 + i_3.$$

Then system (3) can be transformed to the following form

$$\begin{aligned} &\dot y = -cy - \omega_u - x\omega_u,\ \dot x = -cx + y\omega_u,\ \dot z = -cz, \\ &\dot\theta = \omega_l - \omega_u, \\ &\dot\omega_u = \frac{k}{J_u}\theta + \frac{b}{J_u}(\omega_l - \omega_u) + \frac{a}{J_u}y, \\ &\dot\omega_l = -\frac{k}{J_l}\theta - \frac{b}{J_l}(\omega_l-\omega_u) + \frac{1}{J_l}T_{fl}(\omega-\omega_l), \end{aligned} \quad (6)$$

where $a = \frac{3(nSB)^2}{2L}$, $c = \frac{R+r}{L}$, $k = k_\theta$.

Thus, the mathematical model of drilling rig is described by high order differential equations (6) with complicated discontinuous nonlinearity $T_{fl}(\omega-\omega_l)$. These equations are quite hard to study by analytical methods, therefore, here numerical modeling [5] is done in the next section.

[5] The model studied in this work is described by equations with discontinuous right hand-sides, therefore, a special method for numerical computation of their solutions is required. Here the modified event-driven method based on Filippov definition (see Piiroinen and Kuznetsov (2008)) is used for numerical modeling.

4. NUMERICAL STUDIES AND RESULTS

The common technique of spud-in is to run the drill-string through the rotary table which is driven by an induction motor. The drill-string then rotates the bit. Then the bit is lowered into the hole to drill the bedrock. At this moment the drilling system is in idle mode, i.e. there is no friction torque T_{fl} in (6). This system has one stable equilibrium state $y = x = z = \theta = \omega_u = \omega_l = 0$, which corresponds to rotation of both upper and lower discs with the same speed ω without angular displacement. When the teeth on the bit abut the bedrock, the initial load-on occurs. It means that in certain moment friction torque T_{fl} suddenly appears. In this case the following behaviours of the drilling rig are possible: the drilling rig pulls in a new operating mode, it starts oscillating, or it may just get stuck.

Operating mode of the system corresponds to a stable equilibrium state and means that upper and lower disc rotate with the same speed and constant angular displacement. Since the bedrock type is usually changing during the drilling, it is necessary to understand how the drilling rig behaves after such a change. In order for the system to pass from one operating mode into a new operating mode it is necessary that the solution of system (6) with the initial data, corresponding to the previous stable equilibrium state (i.e. previous operation mode), tends to a new stable equilibrium state. However instead of new operating mode hidden stick-slip oscillations may appear during the sudden bedrock change. Fig. 7 illustrates that system (6) has one stable equilibrium and one stable limit cycle. Here the limit cycle represents so-called hidden oscillations since it cannot be detected by the standard simulation, i.e. after the transient processes which starts in the neighborhood of a stable equilibrium.

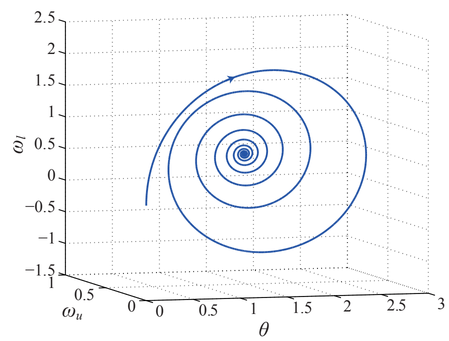

Fig. 6. Stable equilibrium in the mathematical model of drilling system actuated by induction motor without regulation – projection onto $(\theta,\ \omega_u,\ \omega_l)$, $T_0 = 0.25$, $c = 10$

The stick-slip oscillations are determined by rotor vibrations in the drilling rig drive or currents in the rotor circuit. To avoid such oscillations the following strategy is offered. Using the rheostat we reduce the external additional resistance up to motion stabilization. After that the value of external additional resistance is returned to the initial one. In Fig. 8 the trajectory of system (6), falling into the attraction domain of stable limit cycle, is stabilized by suggested strategy. Dashed curve segment A_1A_2 corresponds to the slow change of c from 10 to 1 (motion stabilization). Curve segment A_2A_3 corresponds to the increase of c back

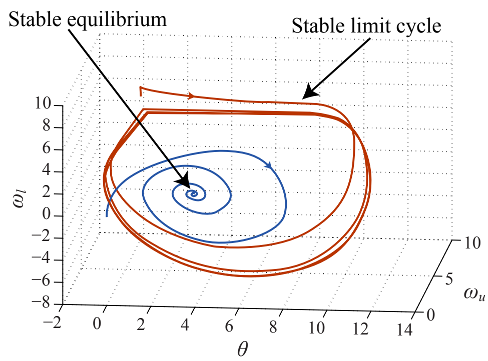

Fig. 7. Hidden oscillations and stable equilibrium in the drilling system actuated by induction motor without regulation – projection onto (θ, ω_u, ω_l), $T_0 = 0.65$, $c = 10$

to the value 10. The same technique can be applied if the drilling system does not go into operating mode during the initial load-on.

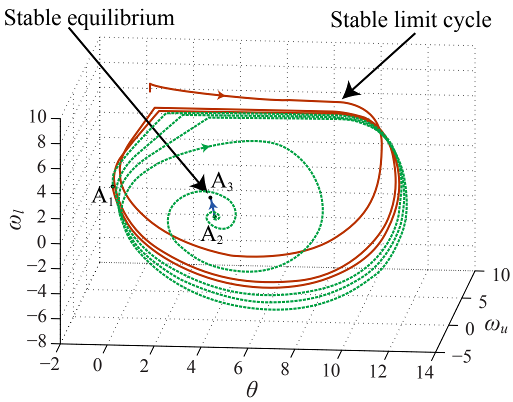

Fig. 8. Stabilization of drilling system actuated by induction motor with regulation – projection onto (θ, ω_u, ω_l), $T_0 = 0.65$, $c \in [1, 10]$

For modeling the following parameters were used: $\omega = 8$, $J_u = 0.4765$, $J_l = 0.035$, $k = 0.075$, $a = 2.1$, $b = 0$, $T_{sl} = 0.26$, $T_{pl} = 0.05$, $\omega_{sl} = 2.2$, $\delta_{sl} = 1.5$, $b_l = 0.009$. [6]

5. CONCLUSION

Drilling string failure is one of the most common problems arising during the drilling processes in the oil and gas industry. Despite a great number of researches devoted to study of drilling rigs, the problem of failure prevention of drilling system still remains unsolved. In this work the mathematical model of a drilling rig driven by an induction motor is presented. During the numerical modeling there were found the so-called hidden oscillations, which may lead to damaging vibrations of the drill-string. Note that the detection of hidden oscillations is a complex task because of their small area of attraction and high dimension of the system. Since such oscillations may lead to drilling systems failures, it is necessary to develop control methods for avoiding these oscillations. Here the simple strategy based on regulation of external additional resistance in the wound rotor circuit is suggested.

[6] In real drilling rigs the speed of rotation of the bit varies between 50 and 300 revolutions per minute (see Short (1983)). The speed of rotation of the bit in idle mode coincide with the synchronous speed of the rotating magnetic field ω, which is defined as $2\pi f/p$, where f is the motor supply frequency, p is the number of pairs of poles (induction motors usually have not less than 8 pairs of poles) (Leonhard, 2001). In our simulation ω is equal to 8 rad/s, which is around 76 resolutions per minute. The speed of rotation of the bit in the operating mode corresponding to the stable equilibrium state depicted e.g. on Fig. 7 is about 62 revolutions per minute.

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